Received: *July 20, 2001*, Accepted: *August 06, 2001*

Bulk quantization of gauge theories: confined and higgs phases

Laurent Baulieu

LPTHE, Universités P. & M. Curie (Paris VI) et D. Diderot (Paris VII), Paris, France
E-mail: baulieu@lpthe.jussieu.fr

Daniel Zwanziger

Physics Department, New York University New-York, NY 10003, USA
E-mail: Daniel.Zwanziger@nyu.edu

Abstract: We deepen the understanding of the quantization of the Yang-Mills field by showing that the concept of gauge fixing in 4 dimensions is replaced in the 5-dimensional formulation by a procedure that amounts to an A-dependent gauge transformation. The 5-dimensional formulation implements the restriction of the physical 4-dimensional gluon field to the Gribov region, while being a local description that is under control of BRST symmetries both of topological and gauge type. The ghosts decouple so the euclidean probability density is everywhere positive, in contradistinction to the Faddeev-Popov method for which the determinant changes sign outside the Gribov region. We include in our discussion the coupling of the gauge theory to a Higgs field, including the case of spontaneously symmetry breaking. We introduce a minimizing functional on the gauge orbit that could be of interest for numerical gauge fixing in the simulations of spontaneously broken lattice gauge theories. Other new results are displayed, such as the identification of the Schwinger-Dyson equation of the five dimensional formulation in the (singular) Landau gauge with that of the ordinary Faddeev-Popov formulation, order by order in perturbation theory.

Keywords: Renormalization Regularization and Renormalons, BRST Quantization, Non-perturbative Effects, QCD.

JHEP08(2001)015

Contents

1. Introduction **1**

2. Step-by-step determination of the TQFT of a gauge theory **4**
- 2.1 Step 1: scalar-field type quantization 4
- 2.2 Step 2: normalization of the path integral by gauge transformation 5
- 2.3 Step 3: introduction of the 5th component A_5 7
- 2.4 Summary: s and w on all fields and 5-dimensional action 9

3. Equivalence to the previous approach and decoupling of ghost fields **10**
- 3.1 Equivalence to the geometrical approach 10
- 3.2 Elimination of ghosts and auxiliary fields 12

4. Confinement and the Gribov region **13**

5. Higgs phase **15**

6. Conclusion **18**

A. Continuation of section 2: steps 4 and 5 **19**
- A.1 Step 4: BRST implementation of the 5-dimensional gauge invariance 19
- A.2 Step 5: construction of w-covariant fields 21

B. Jacobian of gauge transformation **22**

C. Equivalence of standard and bulk quantization for gauge theories **24**

D. Proof of time-reversal invariance **27**

E. Semi-classical analysis of the Higgs phase **28**

1. Introduction

In a previous paper [1], we have described the technique of bulk quantization, which introduces an additional 5-th time, which generalizes stochastic time, and shown the deep relationship between this method and the idea of Topological Field Theories. The delicate symmetries involved by such theories enforce the physical picture that physical observables must be confined to a time-slice of the enlarged space. The BRST symmetry of the theory implies the formal equivalence of Schwinger-Dyson equations in the two formulations. Reference [1] gives a direct definition of the physical S-matrix (assuming that it exists) in the 5-dimensional formulation, together with notion of on-shell particles. Topological invariance ensures the irrelevance of the details of the evolution along the additional time. This previous paper stresses the importance of the symmetry of the theory under reversal of the additional time. The beauty of the construction is quite striking. However, in the case of a scalar theory, one hardly finds advantages for the quantization with an additional time as compared to the ordinary one. In contrast, for gauge theories, conceptual progress does occur. The enlargement of the phase space for off-shell processes solves delicate questions such as the one raised by Gribov a long time ago. In particular the euclidean probability density is everywhere positive, whereas in the 4-dimensional approach the Faddeev-Popov determinant changes sign outside the Gribov region. Our real interest is thus gauge theories, which are the subject of this second paper.

Actually we have in mind the following. According to the ideas of Gribov, solving the question of gauge-fixing in the Yang-Mills theory to reach a definition of the path integral that is valid non-perturbatively is equivalent to inventing a method that confines the integration over the gauge field to a fundamental domain. Doing this implies that the gauge-fixing provides non-trivial and essential information about the gauge field configurations that contribute the functional integral. An analogous situation holds in string theory. There one has a free theory, but the nature of 2D diffeomorphism must be fully accounted for in the gauge-fixing process, including a consistent analysis of moduli transformations. This is known to eventually take into account the full interaction in the theory, although one has "merely" gauge-fixed a free lagrangian. This is an early example where the nature of the interactions is determined by the gauge fixing, that is by geometry in the relevant space. In the Yang-Mills case, the idea of Gribov was that one should find a method to restrict the path integral over the gauge field A to one fundamental domain (with a positive euclidean weight), and moreover that this domain could be chosen in such a way that: (1) A is transverse, and (2) the operator $-\partial_\mu D_\mu(A)$ is positive, that is, all its eigenvalues are positive for every configuration A in the domain [2]. These two conditions define a (larger) region known as "the Gribov region". As a consequence of these conditions, one finds [3] that the gluon propagator $D(k)$ in the Landau gauge cannot exhibit a pole in k at $k = 0$, and in fact $D(k)$ *vanishes* at $k = 0$. Computer simulations have recently been shown to sustain this property [4, 5].) The position of the pole in the transverse part of the gluon propagator is independent of the gauge parameters, by virtue of the Nielsen identities [6], so if this pole is absent in the Landau gauge it is absent in all gauges. The point of view that we adopt in this paper is that bulk quantization is a consistent and operational formulation in 5 dimensions of a quantum field theory in 4 dimensions that automatically satisfies the

Gribov condition. It follows that there can be no free massless gluons in the resulting theory. This is a first and crucial step toward proving confinement.[1] Essential properties such as the existence of a mass gap and bound states remain very difficult, but one may hope that the new framework of bulk quantization will bring new hints for establishing them.

Thus, prior to any investigation of its dynamics, the necessity of a consistent gauge-fixing (in reality, the introduction of a "drift force" tangent to gauge orbits) implies that the massless gluon simply cannot appear in the spectrum — although it plays an essential role as a parton — simply because there is no room non-perturbatively for all Fourier components of an asymptotic massless field within the Gribov region. Further development of Gribov's ideas may be found in [7]. Actually, there have been other attempts to understand confinement from a geometrical point of view [8].

Similar questions arise when gauge theories are coupled coupled to a Higgs field. Perturbatively it seems that the gauge boson can acquire a mass and become part of the physical spectrum. However non-perturbatively there is no clear difference between these two phases because they may be continuously connected, and the status of the gauge boson as an elementary particle or bound state is at issue [9]–[13]. We propose a gauge-fixing appropriate to the Higgs phase in the 5-dimensional formalism which is valid non-perturbatively. It selects a direction for the Higgs field in a way that is consistent with Elitzur's theorem [14]. Moreover it has the advantage that it may be used in lattice simulations of the Higgs phase where it may be implemented by a numerical minimization.

Ideas similar to Gribov's have been developed by Feynman [15] and Singer [16]. They have been implemented in concrete dynamical calculations by Cutkosky and co-workers [17] and by van Baal and co-workers [18] in a hamiltonian formulation of the 4-dimensional theory, keeping a small number of modes. A reasonable hadron spectrum results from the boundary identification of the fundamental modular region, which confirms the validity of Gribov's approach to confinement. However it has proven to be an extremely difficult problem to carry out this program to a higher degree of accuracy precisely because, in the 4-dimensional formalism, the boundary of the fundamental modular region is not provided by the Faddeev-Popov proce-

[1] In Gribov's original formulation, a long-range "force" that confines all colored particles is provided in the Coulomb gauge by the long range of the A_4–A_4 correlator. In the present 5-dimensional formulation it is provided by the A_5–A_5 correlator. The heuristic arguments for confinement in the Coulomb gauge say that the field A_4 carries an infinite range instantaneous anti-screening force. Of course this argument is spoiled by the fact that the Coulomb gauge is not well defined at the non-perturbative level in the Faddeev-Popov formulation because of the existence of Gribov copies which cannot be eliminated by a local action. This problem is overcome in the 5-dimensional formulation, and moreover the field A_5 has engineering dimension 2. This may eventually allow a rigorous proof of confinement by using the instantaneous force in the fifth dimension that is carried by A_5 in the Landau gauge limit.

JHEP08(2001)015

dure, and must be found "by hand", by non-perturbative calculations. On the other hand, as explained in [19], and as discussed in more detail below in section 3, the 5-dimensional formulation automatically restricts the physical 4-dimensional connection to the Gribov region, while being a local gauge theory that is under control of BRST symmetries both of topological and gauge type. This suggests that the Gribov program is truly realizable in the context of a local quantum field theory.

This paper is organized as follows. Section 2, which is continued in appendix A, contains a pedestrian step-by-step construction of the action of the 5-dimensional formulation as an alternative to the geometrical construction given in [19]. It starts from the formalism that we developed in the preceding paper devoted to theories that are not of gauge type [1]. It is shown that the concept of *gauge fixing* is replaced in the 5-dimensional formulation by a procedure that amounts to an A-dependent *gauge transformation*, and one avoids by construction the objection of Singer [16]. We show in appendix B that the jacobian of this gauge transformation is an infinite constant, independent of A, which cancels the divergent volume of the gauge group. In appendix A we introduce the BRST-operator w that codifies the 5-dimensional gauge invariance, with results summarized in section 2. In section 3a we show the equivalence to the previous approach [19], and provide a dictionary that relates the fields introduced here and there. In section 3b we show that the ghost fields decouple because their field equations are parabolic so the ghost propagators are retarded and all ghost loops vanish. In section 4, we indicate how the restoring force along gauge orbits forbids the existence of massless gluons, independently of the details of the confining force. Section 5 is devoted to the case of the coupling of the gauge field to a Higgs field, and we generalize the mechanism that is at work in the confining phase to the Higgs phase. We have relegated other results to appendices. In appendix C, we give a new way of showing the perturbative equivalence of the 4- and 5-dimensional formulations for gauge-invariant quantities. It is an alternative to the old proof [20]. It also explains how the non-pertubatively ill-defined Faddeev-Popov ghost of the 4D formulation can be extracted (in a non-local way) from the well-defined topological ghost of the 5D formulation, in a singular gauge. In appendix D, we establish the invariance of a gauge theory under reversal of the 5th-time, which generalizes the case of a theory of non-gauge type. In appendix E we present a semi-classical treatment of the Higgs mechanism in the 5-dimensional formulation.

2. Step-by-step determination of the TQFT of a gauge theory

2.1 Step 1: scalar-field type quantization

In 4 dimensions, one considers an SU(N) gauge field $A_\mu^a(x)$, with Yang-Mills action $S = S_{\rm YM} = \frac{1}{4}\int d^4x F_{\mu\nu}^2$, where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c$, and $\delta S/\delta A_\mu^a = -(D_\lambda F_{\lambda\mu})^a$. The gauge-covariant derivative is defined by $(D_\mu\omega)^a \equiv \partial_\mu\omega^a + f^{abc}A_\mu^b\omega^c$.

To derive the 5-dimensional formulation in its simplest expression, we start with some results contained in our paper devoted to the case of theories non-gauge type [1] and do the minimal hypothesis that the ungauge-fixed theory corresponds to the pioneering formulation of Parisi and Wu [21], as developed in [22, 23]. (This first step is formal because of divergences associated with the infinite volume of the gauge group.) It prescribes that, corresponding to the 4-dimensional euclidean field $A_\mu(x)$, is the quartet $A_\mu(x,t), \psi_\mu(x,t), \bar\psi_\mu(x,t), \pi_\mu(x,t)$ on which a topological BRST-operator acts according to

$$\begin{aligned} sA_\mu &= \psi_\mu\,, \qquad & s\psi_\mu &= 0\,, \\ s\bar\psi_\mu &= \pi_\mu\,, \qquad & s\pi_\mu &= 0\,, \end{aligned} \tag{2.1}$$

and, moreover, corresponding to the classical Yang-Mills action $S_{\rm YM}$ is the BRST-exact bulk action

$$I_{\rm YM} \equiv \int dt\ d^4x\ s\left[\bar\psi_\mu\left(\partial_t A_\mu + \frac{\delta S_{\rm YM}}{\delta A_\mu} + \pi_\mu\right)\right]. \tag{2.2}$$

We have $\delta S_{\rm YM}/\delta A_\mu = -D_\lambda F_{\lambda\mu}$, where $F_{\lambda\mu} = \partial_\lambda A_\mu - \partial_\mu A_l + [A_\lambda, A_\mu]$ is the Yang-Mills field tensor. The field π_μ (which would be written b_μ in the notation of [1]) is the momentum density canonical to A_μ in the 5-dimensional theory. Indeed, upon expansion we obtain

$$\begin{aligned} I_{\rm YM} = \int dt\ d^4x\ \big[&\pi_\mu\left(\partial_t A_\mu - D_\lambda F_{\lambda\mu} + \pi_\mu\right) - \\ &- \bar\psi_\mu\left(\partial_t\psi_\mu - D_\lambda(D_\lambda\psi_\mu - D_\mu\psi_\lambda) - [\psi_\lambda, F_{\lambda\mu}]\right)\big]\,, \end{aligned} \tag{2.3}$$

and we have $\pi_\mu = \partial\mathcal{L}/\partial\dot A_\mu$, where $\dot A_\mu \equiv \partial_t A_\mu$. Similarly we have $\bar\psi_\mu = \partial\mathcal{L}/\partial\dot\psi_\mu$, where the left-derivative is taken. A consequence of this action is that on a given time slice the formal ungauge-fixed Schwinger-Dyson equations of $S_{\rm YM}$ are satisfied, as in theory of non-gauge type [1].

2.2 Step 2: normalization of the path integral by gauge transformation

The action $I_{\rm YM}$ inherits from $S_{\rm YM}$ invariance under local 4-dimensional gauge transformations $g(x)$ under which the fields transform according to

$$\begin{aligned} A_\mu &\longrightarrow {}^gA_\mu = g^{-1}A_\mu g + g^{-1}\partial_\mu g\,, \\ \psi_\mu &\longrightarrow {}^g\psi_\mu = g^{-1}\psi_\mu g\,, \\ \bar\psi_\mu &\longrightarrow {}^g\bar\psi_\mu = g^{-1}\bar\psi_\mu g\,, \\ \pi_\mu &\longrightarrow {}^g\pi_\mu = g^{-1}\pi_\mu g\,. \end{aligned} \tag{2.4}$$

Consequently the action $I_{\rm YM}$ provides no convergence for the longitudinal modes. We must cure this problem.

Consider now a local gauge transformation that is also t-dependent, $g = g(x,t)$. This is clearly a symmetry transformation for gauge-invariant observables $\mathcal{O}({}^gA) = \mathcal{O}(A)$, in particular for those that depend on $A_\mu(x,t)$ at a fixed time, $A_\mu(x,0)$ say.[2] However the action I_{YM} is not invariant under transformations $g(x,t)$ because of the terms involving time derivatives. They transform according to

$$\begin{aligned}
\partial_t A_\mu &\longrightarrow \partial_t \left({}^gA_\mu\right) = g^{-1}\left(\partial_t A_\mu - D_\mu v\right) g\,, \\
\partial_t \psi_\mu &\longrightarrow \partial_t \left({}^g\psi_\mu\right) = g^{-1}\left(\partial_t \psi_\mu - [\psi_\mu, v]\right) g\,,
\end{aligned} \tag{2.5}$$

where $[X,Y]^a \equiv f^{abc}X^bY^c$, and $v \equiv -\partial_t g g^{-1} = g\partial_t g^{-1}$. Under this gauge transformation I_{YM} becomes

$$\hat{I}_{\mathrm{YM}} = \int dt\ d^4x\ s\left[\bar{\psi}_\mu\left(\partial_t A_\mu - D_\mu v - D_\lambda F_{\lambda\mu} + \pi_\mu\right)\right]. \tag{2.6}$$

This action is physically equivalent to I_{YM}. Moreover given any $v(x,t)$, we may solve for $g(x,t)$, so $v(x,t)$ is a function at our disposal. We shall in fact choose

$$v = a^{-1}\partial_\mu A_\mu\,, \tag{2.7}$$

as was also made by [24, 25, 26]. With this choice, that is actually compulsory if renormalizability by power counting is required in the five dimensional quantum field theory, the action $\hat{I}_{\mathrm{YM}}$ provides convergence for all modes including the longitudinal modes, as we shall see. In doing so we are merely choosing a gauge *transformation*, $v = -\partial_t g g^{-1}$, but no gauge *fixing* is done so the issue of Gribov copies does not arise.[3] Once v is determined, so is its s-transform sv. The transformed action with $v = a^{-1}\partial_\mu A_\mu$ is given after expansion by

$$\begin{aligned}
\hat{I}_{\mathrm{YM}} = \int dt\ d^4x\ \Big[&\pi_\mu\left(\partial_t A_\mu - a^{-1}D_\mu\partial_\lambda A_\lambda - D_\lambda F_{\lambda\mu} + \pi_\mu\right) - \\
&- \bar{\psi}_\mu\big(\partial_t\psi_\mu - a^{-1}D_\mu\partial_\lambda\psi_\lambda - a^{-1}[\psi_\mu, \partial_\lambda A_\lambda] - \\
&\qquad - D_\lambda\left(D_\lambda\psi_\mu - D_\mu\psi_\lambda\right) - [\psi_\lambda, F_{\lambda\mu}]\big)\Big].
\end{aligned} \tag{2.8}$$

To see that this action provides convergence for the longitudinal modes consider its quadratic part,

$$\begin{aligned}
(\hat{I}_{\mathrm{YM}})_0 = \int dt\ d^4x\ \Big[&\pi_\mu\left(\partial_t A_\mu - a^{-1}\partial_\mu\partial_\lambda A_\lambda - \partial_\lambda(\partial_\lambda A_\mu - \partial_\mu A_\lambda) + \pi_\mu\right) - \\
&- \bar{\psi}_\mu\left(\partial_t\psi_\mu - a^{-1}\partial_\mu\partial_\lambda\psi_\lambda - \partial_\lambda\left(\partial_\lambda\psi_\mu - \partial_\mu\psi_\lambda\right)\right)\Big].
\end{aligned} \tag{2.9}$$

[2] For a pure gauge theory without quarks, the time t has a stochastic interpretation and corresponds to the number of sweeps in a Monte Carlo calculation. The gauge transformation $g(x,t)$ corresponds to making a gauge transformation after each sweep. The functional dependence of $g(x,t)$ on A corresponds to choosing $g(x,t)$ to depend on $A(x,t)$, as is common practice when $g = g[A]$ is chosen by a minimization process.

[3] This was pointed out many years ago in the context of the Langevin equation [24]. Here we obtain the same result by the more conventional functional integral methods.

From it we obtain the free propagators in momentum space

$$\begin{aligned} D^{AA,\mathrm{tr}} &= 2\left[\omega^2 + (k^2)^2\right]^{-1}, & D^{AA,\mathrm{long}} &= 2\left[\omega^2 + a^{-2}\,(k^2)^2\right]^{-1}, \\ D^{\psi\bar\psi,\mathrm{tr}} &= \left[i\omega + k^2\right]^{-1}, & D^{\psi\bar\psi,\mathrm{long}} &= \left[i\omega + a^{-1}k^2\right]^{-1}, \end{aligned} \tag{2.10}$$

and by s-invariance $D^{Ab}_{\lambda\mu} = -D^{\psi\bar\psi}_{\lambda\mu}$. The parameter a^{-1} provides convergence of the longitudinal modes, as asserted, and $a = 0$ is the Landau-gauge limit. We take the parameter $a^{-1} > 0$.

It may seem paradoxical that regularization of the longitudinal modes, which requires division by the infinite volume of the gauge orbit, has been be achieved by a gauge transformation, which is the interpretation that we gave in this section. The answer to this is that gauge transformations g leave the functional measure $\mathcal{D}A$ invariant only when g is independent of the variable A. However if $g = g[A]$ is a functional[4] of A, then the jacobian J of the transformation $A \to {}^gA$ is not necessarily unity. In appendix B, we will calculate J for the transformation with $v = -\partial_t g g^{-1} = a^{-1}\partial_\mu A_\mu$, and find that it is an infinite constant, independent of A. This is an essential point, for if the jacobian of the regularizing gauge transformation were A-dependent, $J = J[A]$, there would be additional corrections to the action. Clearly the mechanism of regularization by gauge transformation is quite different from ordinary gauge fixing which requires choosing a point on each gauge orbit, and for this reason the Gribov problem does not arise.

2.3 Step 3: introduction of the 5th component A_5

If one introduces the notation

$$x_5 \equiv t\,, \qquad A_5 \equiv v\,, \qquad \psi_5 \equiv sA_5\,, \tag{2.11}$$

one recognizes that $\partial_t A_\mu - D_\mu v = \partial_5 A_\mu - \partial_\mu A_5 - [A_\mu, A_5] = F_{5\mu}$ is a component of the Yang-Mills field tensor in 5 dimensions. Regarded as a function of the new variables, the action $\hat I_{\mathrm{YM}}(A_\mu, A_5, \psi_\mu, \psi_5, \bar\psi_\mu, \pi_\mu)$ reads

$$\hat I_{\mathrm{YM}} \equiv \int d^5x\, s\left[\,\bar\psi_\mu\,(F_{5\mu} - D_\lambda F_{\lambda\mu} + \pi_\mu)\right]. \tag{2.12}$$

It is manifestly invariant under the 5-dimensional gauge transformation $g(x,t)$ that depends both on x and t, under which the fields transform according to (2.4) supplemented by

$$\begin{aligned} A_5 &\longrightarrow {}^gA_5 = g^{-1}A_5 g + g^{-1}\partial_5 g\,, \\ \psi_5 &\longrightarrow {}^g\psi_5 = g^{-1}\psi_5 g\,. \end{aligned} \tag{2.13}$$

[4]The transformations (2.4) of ψ_μ and π_μ hold only when g is independent of A. Otherwise they are given by ${}^g\psi_\mu = s\,{}^gA_\mu$ and ${}^g\pi_\mu = s\,{}^g\bar\psi_\mu$.

We have seen that we may regularize the longitudinal modes by choosing

$$A_5 = a^{-1}\partial_\nu A_\nu \,, \tag{2.14}$$

without encountering the Gribov problem for the physical variables A_μ, with $\mu = 1, \ldots, 4$. The new notation reveals that the regularization of the 4-dimensional gauge invariance by gauge transformation resembles a linear gauge-fixing condition of the 5-dimensional gauge symmetry by $aA_5 = \partial_\mu A_\mu$. This relation implies

$$\psi_5 = a^{-1}\partial_\nu \psi_\nu \,. \tag{2.15}$$

Note that A_5 itself appears in the gauge condition rather than its derivative $\partial_5 A_5$. In this respect the gauge-fixing of the 5-dimensional theory resembles the axial gauge for which the ghosts decouple because, as we shall see, the Faddeev-Popov determinant is known to be trivial. However whereas the axial gauge in 4 dimensions is ambiguous, the 5-dimensional theory is well defined and renormalizable. Moreover this gauge condition does not violate Singer's theorem because t extends over an infinite interval whereas Singer's theorem applies to compact space-time [16]. The gain over the conventional Faddeev-Popov formulation is enormous because the ghosts decouple, as will be shown below. As a result, the euclidean weight, after elimination of auxiliary fields, is positive everywhere. By contrast, in the 4-dimensional approach the Faddeev-Popov determinant changes sign outside the Gribov region.

We automate the conditions $aA_5 = \partial_\mu A_\mu$ and $a\psi_5 = \partial_\mu \psi_\mu$ by adding an action

$$\hat{I}_{\rm gf} \equiv \int d^5x \; [l\,(aA_5 - \partial_\nu A_\nu) - \bar{m}\,(a\psi_5 - \partial_\nu \psi_\nu)] \,. \tag{2.16}$$

that contains two Lagrange multiplier fields l and $\bar{m}$, one for each of these the conditions. To maintain s-invariance and to keep s trivial, in the sense that it acts on an elementary field to produce another elementary field rather than a composite field, we arrange the new variables and their Lagrange multipliers into a quartet $(A_5, \psi_5, \bar{m}, l)$, like $(A_\mu, \psi_\mu, \bar{\psi}_\mu, \pi_\mu)$, within which s acts according to

$$\begin{aligned} sA_5 &= \psi_5 \,, \qquad & s\psi_5 &= 0 \,, \\ s\bar{m} &= l \,, \qquad & sl &= 0 \,, \end{aligned} \tag{2.17}$$

as in (2.1). (We use the notation $\bar{m}$ and l — rather than $\bar{\psi}_5$ and π_5 — for these Lagrange multiplier fields that enforce time-independent constraints, to distinguish them from the four $\bar{\psi}_\mu$ and π_μ, that impose time-dependent equations of motion.) The new action may be written in the s-exact form

$$\hat{I}_{\rm gf} = \int d^5x \; s\,[\bar{m}\,(aA_5 - \partial_\nu A_\nu)] \,, \tag{2.18}$$

and the total action

$$\begin{aligned}
\hat{I} &\equiv \hat{I}_{\rm YM} + \hat{I}_{\rm gf} \\
&= \int d^5x \; s \left[\bar{\psi}_\mu \left(F_{5\mu} - D_\lambda F_{\lambda\mu} + \pi_\mu \right) + \bar{m} \left(a A_5 - \partial_\nu A_\nu \right) \right]
\end{aligned} \tag{2.19}$$

is equivalent to the action (2.8). As it stands, this action does not provide easy access to the Ward identities that express the 5-dimensional gauge invariance of $\hat{I}_{\rm YM}$. This will be done by the introduction of a second BRST operator w that encodes gauge invariance.

2.4 Summary: s and w on all fields and 5-dimensional action

Steps 4 and 5 are somewhat lengthy and are consigned to appendix A. We summarize here the results of steps 4 and 5. There are two BRST operators: the s-operator, introduced above, that is topological, and the w-operator that encodes gauge invariance. They are algebraically consistent in the sense that $s^2 = w^2 = sw + ws = 0$.

The action of s and of w on all fields is given by

$$\begin{aligned}
&sA_\nu = \psi_\nu \,, \qquad s\psi_\nu = 0 \,, \qquad s\bar{\psi}_\nu = \pi_\nu \,, \qquad s\pi_\nu = 0 \,, \\
&sA_5 = \psi_5 \,, \qquad s\psi_5 = 0 \,, \qquad s\bar{m} = l \,, \qquad sl = 0 \,, \\
&s\lambda = \mu \,, \qquad s\mu = 0 \,, \qquad s\bar{\mu} = \bar{\lambda} \,, \qquad s\bar{\lambda} = 0 \,, \\
&s\omega = \phi \,, \qquad s\phi = 0 \,, \qquad s\bar{\phi} = \bar{\omega} \,, \qquad s\bar{\omega} = 0 \,;
\end{aligned} \tag{2.20}$$

$$\begin{aligned}
&wA_\nu = D_\nu \lambda \,, \qquad w\psi_\nu = -[\lambda, \psi_\nu] - D_\nu \mu \,, \qquad w\bar{\psi}_\nu = -[\lambda, \bar{\psi}_\nu] \,, \\
&w\pi_\nu = -[\lambda, \pi_\nu] + [\mu, \bar{\psi}_\nu] \,, \qquad wA_5 = D_5 \lambda \,, \qquad w\psi_5 = -[\lambda, \psi_5] - D_5 \mu \,, \\
&w\bar{m} = 0 \,, \qquad wl = 0 \,, \qquad w\lambda = -\frac{1}{2}[\lambda, \lambda] \,, \qquad w\mu = -[\lambda, \mu] \,, \qquad w\bar{\mu} = \bar{m} \,, \\
&w\bar{\lambda} = -l \,, \qquad w\omega = -[\lambda, \omega] - \mu \,, \qquad w\phi = -[\lambda, \omega] + [\mu, \omega] \,, \qquad w\bar{\phi} = -[\lambda, \bar{\phi}] \,, \\
&w\bar{\omega} = -[\lambda, \bar{\omega}] + [\mu, \bar{\phi}] \,.
\end{aligned} \tag{2.21}$$

The algebra of s and w closes on the fields of the first three lines. The last quartet is not needed for algebraic consistency but is needed to construct an action that is both s- and w-invariant.

Associated to the symmetry generators s and w are independently conserved ghost numbers N_s and N_w which are increased by unity by the action of s and w. We make the following ghost number assignment, indicated by the superscripts (N_s, N_w), consistent with this and with (2.20) and (2.21):

$$\begin{array}{ccccccccc}
 & & \bar{\psi}_\nu^{(-1,0)} & \longrightarrow & \pi_\nu^{(0,0)} A_\nu^{(0,0)} & \longrightarrow & \psi_\nu^{(1,0)} & & \\
 & & \bar{m}^{(-1,0)} & \longrightarrow & l^{(0,0)} A_5^{(0,0)} & \longrightarrow & \psi_5^{(1,0)} & & \\
 & \nearrow & & \nearrow & & & & & \\
\bar{\mu}^{(-1,-1)} & \longrightarrow & \bar{\lambda}^{(0,-1)} & & & & \lambda^{(0,1)} & \longrightarrow & \mu^{(1,1)} \\
\bar{\phi}^{(-2,0)} & \longrightarrow & \bar{\omega}^{(-1,0)} & & & & \omega^{(1,0)} & \longrightarrow & \phi^{(2,0)} \,.
\end{array} \tag{2.22}$$

Each column corresponds to a fixed value of the total ghost number $N \equiv N_s + N_w$. Fields with even N are bosonic; otherwise they are fermionic. Each row corresponds to a single quartet within which s acts as indicated by the horizontal arrows $\rightarrow$. The two northeast arrows $\nearrow$ indicate the action of w, but only where it produces the elementary Lagrange multiplier fields $\bar{m}$ and $-l$. (Otherwise w produces a composite field.) This is the minimum number of fields that is required to construct an action that is both s- and w-invariant and that is physically equivalent to the preceding action.

We also introduce the composite fields,

$$\pi_\nu^\star \equiv \pi_\nu + [\omega, \bar{\psi}_\nu]\,, \tag{2.23}$$

$$\psi_\mu^\star \equiv \psi_\mu - D_\mu\omega\,, \qquad \psi_5^\star \equiv \psi_5 - D_5\omega\,, \tag{2.24}$$

that are w-covariant, $w\pi_\nu^\star = -[\lambda, \pi_\nu^\star]$. $w\psi_\mu^\star = -[\lambda, \psi_\mu^\star]$, $w\psi_5^\star = -[\lambda, \psi_5^\star]$.

A consistent bulk action for gauge fields is provided by

$$\begin{aligned} I \equiv \int d^5x\, \big[s\,\big(\bar{\psi}_\mu(F_{5\mu} - D_\lambda F_{\lambda\mu} + \pi_\mu + [\omega, \bar{\psi}_\mu]\big) \; + \\ + \bar{\phi}[a'(\psi_5 - D_5\omega) - D_\mu(\psi_\mu - D_\mu\omega)]\big) + sw(\bar{\mu}(aA_5 - \partial_\nu A_\nu))\big]\,. \end{aligned} \tag{2.25}$$

It is s-exact and w-invariant, $wI = 0$. All terms except the last are in the cohomology of w, and the last term is w-exact. The expansion of the various terms in this action is given in appendix A.

3. Equivalence to the previous approach and decoupling of ghost fields

3.1 Equivalence to the geometrical approach

Remarkably, the action and fields that have just been derived agree precisely with the corresponding quantities of [19] which was obtained by quite different geometrical reasoning. These fields were displayed in the following pyramidal diagram:

$$\begin{tikzcd}
 & & A_\mu^{(00)}, A_5^{(00)} \arrow[dl] & & \\
 & \begin{matrix} \Psi_\mu^{(1,0)}, \Psi_5^{(1,0)} \\ c^{(1,0)}, \lambda^{(0,1)} \end{matrix} \arrow[dl] & & \begin{matrix} \bar{\Psi}_\mu^{(-1,0)}\,, \\ \bar{c}^{(-1,0)}\bar{\lambda}^{(0,-1)} \end{matrix} \arrow[dl] & \\
\begin{matrix} \Phi^{(2,0)}, \\ \mu^{(1,1)} \end{matrix} & & \begin{matrix} b_\mu^{(0,0)} \\ l^{(0,0)} \end{matrix} & & \begin{matrix} \bar{\Phi}^{(-2,0)}\,, \\ \bar{\mu}^{(-1,-1)} \end{matrix} \arrow[ul] \arrow[dl] \\
 & & & \bar{\eta}^{(-1,0)}\,, &
\end{tikzcd} \tag{3.1}$$

To exhibit the correspondence between these fields and the ones in the present article requires a non-linear field redefinition to provide fields that transform gauge-covariantly under w. For this purpose we also need "adjusted" fields $\phi^\star$ and $\bar\omega^\star$ that transform gauge-covariantly,

$$\begin{aligned} \phi^\star &\equiv \phi + \frac{1}{2}[\omega,\omega]\,, \qquad & w\phi^\star &= -[\lambda,\phi^\star]\,,\\ \bar\omega^\star &\equiv \bar\omega + [\omega,\bar\phi]\,, \qquad & w\bar\omega^\star &= -[\lambda,\bar\omega^\star]\,. \end{aligned} \tag{3.2}$$

The correspondences are given by

$$\begin{aligned} A_\nu &= A_\nu\,; \qquad & \Psi_\nu &= \psi^\star_\nu = \psi_\nu - D_\nu\omega\,,\\ \bar\Psi_\nu &= \bar\psi_\nu\,; & b_\nu &= \pi^\star_\nu = \pi_\nu + [\omega,\bar\psi_\nu]\,,\\ A_5 &= A_5\,; & \Psi_5 &= \psi^\star_5 = \psi_5 - D_5\omega\,,\\ \bar c &= \bar m\,; & l &= l\,,\\ \lambda &= \lambda\,; & \mu &= \mu\,; \qquad \bar\mu = \bar\mu\,; \qquad \bar\lambda = \bar\lambda\,,\\ c &= \omega\,; & \Phi &= \phi^\star \equiv \phi + \left(\frac{1}{2}\right)[\omega,\omega]\,,\\ \bar\Phi &= \bar\phi\,; & \bar\eta &= \bar\omega^\star \equiv \bar\omega + [\omega,\bar\phi]\,, \end{aligned} \tag{3.3}$$

(where upper and lower cases are distinguished). When expressed in terms of the new variables, the action (2.25) is the action of [19] where its renormalizability and other properties are established. Because the field redefinition $\Psi_5 = \psi^\star_5 = \psi_5 - D_5\omega$ involves a time derivative, dynamical and non-dynamical field equations become interchanged in the action, so the non-dynamical Lagrange multiplier $\bar m$ becomes the dynamical Lagrange multiplier $\bar c$ and conversely for $\bar\omega$ and $\bar\eta$.

Consistency of the construction is revealed by the geometrical formula:

$$\begin{aligned} (d+s+w)(A+c+\lambda) + \frac{1}{2}[A+c+\lambda, A+\omega+\lambda] &= F+\Psi+\Phi\,,\\ (d+s+w)(F+\Psi+\Phi) + [A+c+\lambda, F+\Psi+\Phi] &= 0\,,\\ (s+w)\bar\Phi + [c+\lambda,\bar\Phi] &= \bar\eta\,,\\ (s+w)\bar\eta + [c+\lambda,\bar\eta] &= [\Phi,\bar\Phi]\,,\\ (s+w)\bar\mu &= \bar c + \bar\lambda\,,\\ (s+w)(\bar c+\bar\lambda) &= 0 \end{aligned} \tag{3.4}$$

which implies that one has $(s+w)^2 = 0$ by construction. Equation (3.4) is typical of a topological gauge symmetry. Fields like μ and l are introduced to solve the degeneracy of the equation $s\lambda + w\omega + [\omega,\lambda] = 0$ and $s\bar c + w\bar\omega = 0$. The conservation of both ghost numbers N_s and N_w is of course most important in this determination.

As explained in [19], the s-invariance enforces the possibility of defining observables in any given slice, the w-invariance expresses the Yang-Mills gauge symmetry of the theory. Actually, observables are defined as the cohomology of w, that one can restrict to a slice, provided no anomaly occurs. Actually, power counting and the requirements of locality, s and w invariances, (5th) time parity symmetry and ghost number conservation completely determine the local five-dimensional action I, eq. (2.25).

3.2 Elimination of ghosts and auxiliary fields

We now show that the action I is physically equivalent to the original action (2.19). The argument relies on the fact that all ghosts decouple because all ghost propagators are retarded and ghost numbers are conserved. Indeed all free ghost propagators such as (2.10) are analytic in the lower half ω-plane. Consequently all the free ghost propagators are retarded, $D_{\lambda\mu}^{\psi\bar{\psi}}(x,t) = 0$ for $t < 0$ and likewise for the other ghost propagators. This is a characteristic of parabolic field equations. Since every ghost propagator is retarded, *all closed ghost loops vanish.*[5] This property is essential to the 5-dimensional formulation of physical 4-dimensional gauge theories. Moreover we may start at any ghost line with non-zero N_w in a Feynman diagram and follow the conserved ghost charge N_w into the future where it becomes an external N_w-ghost line. Therefore each diagram with no external N_w-ghosts has no internal N_w-ghost line either. Consequently integration over the N_w-ghosts results simply in the suppression of these ghosts in the action. This argument remains valid despite the presence of triple ghost vertices such as $\bar{\mu}[\lambda, \psi_5]$ in the action I. A similar argument also allows us to separately integrate out the fields of the last quartet $(\omega, \phi, \bar{\phi}, \bar{\omega})$. The action (2.19) results. This also shows that the expectation-values of physical observables is independent of the parameter a' since it does not appear in (2.19).

The above argument also holds for integration over ghost fields with $N_s \neq 0$ but $N_w = 0$. As a result, for computing correlation functions for which all external lines have ghost number $N_s = 0$, but possibly $N_w \neq 0$, we may suppress all ghost fields with ghost number $N_s \neq 0$ in the action (2.25), which gives

$$I_{\rm w}(a) = \int d^5x \left(\pi_\mu^\star \left(F_{5\mu} + \frac{\delta S}{\delta A_\mu} + \pi_\mu^\star \right) - w[\bar{\lambda}(aA_5 - \partial_\mu A_\mu)] \right). \tag{3.5}$$

Here we have written $-D_\lambda F_{\lambda\mu} = \delta S\ /\delta A_\mu$, and used the w-covariant field variable $\pi_\mu^\star$ defined in (2.23). We will use this action in appendix D to show invariance of physical observables under inversion of the 5th time. The expectation-value of physical observables is independent of the gauge parameter a because they are in the cohomology of w, but the gauge-parameter a appears only in the w-exact term in the actions (2.25) or (3.5).

It also follows that the set of correlation functions with no external ghost lines — and this includes all physical correlation functions — contains no internal ghost lines either. Consequently this set of correlation functions is described by the action (2.25)

[5]There are ghost tadpole diagrams that we neglect. They serve only to cancel other tadpole diagrams, and vanish with dimensional regularization.

in which all ghost fields are suppressed namely the reduced action

$$I_{\rm red} = \int dt\ d^4x\ \pi_\mu \left(\partial_t A_\mu - a^{-1} D_\mu \partial_\lambda A_\lambda - D_\lambda F_{\lambda\mu} + \pi_\mu\right). \tag{3.6}$$

If we also integrate over the π_μ field (it is purely imaginary) we obtain the completely reduced action

$$I'_{\rm red} = -\left(\frac{1}{4}\right) \int dt\ d^4x\ \left(\partial_t A_\mu - a^{-1} D_\mu \partial_\lambda A_\lambda - D_\lambda F_{\lambda\mu}\right)^2, \tag{3.7}$$

and positive euclidean weight $\exp(I'_{\rm red})$ (with $I'_{\rm red} < 0$). Upon taking the square in the last expression, the cross-terms involving $-D_\lambda F_{\lambda\mu} = \delta S/\delta A_\mu$ are exact derivatives. Indeed by the gauge-invariance of S we have $\int d^4x\ \partial_\lambda A_\lambda D_\mu \frac{\delta S}{\delta A_\mu} = 0$ and also $\int d^4x\ \partial_t A_\mu \frac{\delta S}{\delta A_\mu} = \dot{S}$. Thus we may define

$$\begin{aligned} I'_{\rm tot} &\equiv -\int dt \left(\frac{1}{2}\right) \dot{S} + I'_{\rm red}\,, \\ I'_{\rm tot} &= -\left(\frac{1}{4}\right) \int dt\ d^4x [(\partial_t A_\mu - a^{-1} D_\mu \partial_\lambda A_\lambda)^2 + (D_\lambda F_{\lambda\mu})^2]\,, \end{aligned} \tag{3.8}$$

which is a sum of squares. When this weight is used in the path integral over $\mathcal{D}A_{x,t}$ the difficulties that Faddeev-Popov distribution in 4 dimensions encounters at the non-perturbative level are avoided.

4. Confinement and the Gribov region

We have seen that use of the fifth dimension avoids the formal difficulties of gauge-fixing in 4 dimensions that is problematical at the non-perturbative level. We shall now show explicitly how the resulting local 5-dimensional action in fact concentrates the weight in or near the Gribov region. This can only be achieved in the 4-dimensional formulation by topological identification of boundary of the fundamental region, which requires difficult non-perturbative calculations [17, 18]. Here we recall and sharpen the discussion of [19] of the limit $a \to 0$ of the path integral (3.8). (Recall that the mean values of observables are independent of the choice the gauge parameter a.)

Upon rescaling the time according to $t \to at$, we obtain

$$I'_{\rm tot} = -\left(\frac{1}{4}\right) \int dt\ d^4x [a^{-1}(\partial_t A_\mu - D_\mu \partial_\lambda A_\lambda)^2 + a\ (D_\lambda F_{\lambda\mu})^2]\,, \tag{4.1}$$

In the limit $a \to 0$, the path integral over $A_\mu(x,t)$ gets concentrated near where the condition $F_{5\mu} = \partial_t A_\mu - D_\mu \partial_\lambda A_\lambda = 0$, is satisfied, namely near configurations that satisfy the flow equation

$$\partial_t A_\mu = D_\mu \partial_\lambda A_\lambda\,. \tag{4.2}$$

JHEP08(2001)015

We now make a global analysis of this flow. The velocity field $D_\mu \partial_\lambda A_\lambda$ is an infinitesimal gauge transformation, with generator $\omega = \partial_\lambda A_\lambda$, so the flow at each point $A = A_\mu(x)$ is tangent to the gauge orbit through A. We assert that the flow (4.2) is, at each point A, in the direction of steepest descent of the "minimizing" functional,

$$\mathcal{F}_A[g] = \mathcal{F}_{^gA}[1] \equiv ||^gA||^2 \,, \tag{4.3}$$

defined on the gauge orbit through A. Here $||A||^2 \equiv \int d^4x |A_\mu|^2$ is the 4-dimensional Hilbert-norm, and $^gA_\mu = g^{-1}A_\mu g + g^{-1}\partial_\mu g$ is the gauge-transform of A_μ. To prove the assertion, consider the variation of the functional $\mathcal{F}_A[1] \equiv ||A||^2$ under an arbitrary infinitesimal gauge transformation, $\delta A_\mu = D_\mu(A)\omega$,

$$\delta||A||^2 = 2(A_\mu, \delta A_\mu) = 2(A_\mu, D_\mu \omega) = 2(A_\mu, \partial_\mu \omega) = -2(\partial_\mu A_\mu, \omega) \,. \tag{4.4}$$

Thus the direction of steepest descent of $||A||^2$, restricted to directions tangent to the gauge orbit at A, is given by the generator $\omega = \partial_\mu A_\mu$, which is what we wished to establish. Starting from an arbitrary configuration, the flow (4.2), $||A||^2$ decreases monotonically, $\partial_t||A||^2 = -2||\partial_\mu A_\mu||^2$. Because $||A||^2$ is bounded below, we have $\lim_{t\to\infty} ||\partial_\mu A_\mu||^2 = 0$. This implies $\lim_{T\to\infty} T^{-1} \int_0^T dt||\partial_\mu A_\mu||^2 = 0$, or, in words, over an infinite time interval the time average of $||\partial_\mu A_\mu||^2$ vanishes. Non-zero values of $\partial_\mu A_\mu$ are mere transients, and under the flow (4.2), during an infinite time interval, the weight is entirely concentrated on transverse configurations $A^{\rm tr}$ that satisfy Landau-gauge condition $\partial_\lambda A^{\rm tr}_\lambda = 0$.

By eq. (4.4), these are the stationary points of the minimizing functional $F_A[g] = ||^gA||^2$ at $g = 1$. Stationary points may be either minima or saddle-points, according as the second variation of the minimizing functional $\delta^2 F[A]$, restricted to directions $\delta A_\mu = D_\mu(A)\omega$ tangent to the gauge orbit through A, is positive for all ω or not. From (4.4), it is given by

$$\delta^2 F[A] = -2\delta(\omega, \partial_\mu A_\mu) = -2(\omega, \partial_\mu D_\mu(A)\omega) \,. \tag{4.5}$$

Only exceptional configurations flow to saddle-points, where the equilibrium under the "force" $D_\mu \partial_\lambda A_\lambda$ is unstable. (For if one moves off a point of unstable equilibrium, then in general one picks an unstable component of the force.) Non-exceptional configurations flow to minima, which are stable attractors. We conclude that under the flow (4.2), and for an infinite-time interval, the weight is entirely concentrated on configurations that satisfy two conditions: (i) they are transverse $\partial_\lambda A_\lambda = 0$ and (ii) the Faddeev-Popov operator is positive $-\partial_\mu D_\mu(A) > 0$. These two conditions define the Gribov region. Thus in the limit $a \to 0$, the weight corresponding to the partition function (4.1) gets concentrated inside the Gribov region. For small positive values of the gauge parameter a, the weight is smeared out on each gauge orbit but concentrated near the Gribov region.

The first condition, transversality, is a standard gauge-fixing condition of the 4-dimensional formulation. However the second condition, the positivity of the Faddeev-Popov operator, is not achievable by a local 4-dimensional action. It was shown in [3] that the gluon propagator $D(k)$ vanishes at $k = 0$ for a probability distribution concentrated in the Gribov region. As was discussed in the Introduction, this excludes the possibility of a pole at $k^2 = 0$ which corresponds to a physical massless gluon. Since poles of the propagator are independent of the gauge parameter a by virtue of the Nielsen identities [6], this conclusion holds for all values of a. Absence of a massless gluon pole is an important first step toward proving confinement.

5. Higgs phase

We now show how the above considerations may be extended to the Georgi-Glashow or the standard model. For simplicity we take the Georgi-Glashow model, with classical 4-dimensional euclidean action

$$S = \int d^4x \left[\left(\frac{1}{4}\right) F_{\mu\nu}^2 + \left(\frac{1}{2}\right) (D_\mu \phi)^2 + \left(\frac{1}{4}\right) \lambda(\phi^2 - v^2)^2 \right], \tag{5.1}$$

where $\phi = (\phi^a) = \vec{\phi}$ is in the adjoint representation of the SU(2) group. We shall show how the 5-dimensional formulation allows a global analysis of gauge fixing in the Higgs phase, as in the pure gauge case. This is especially important because in this model, the perturbative and the exact spectrum do not agree. Indeed as is well known, a semi-classical analysis results in spontaneous breaking of the SU(2) symmetry with a massless "photon" associated with the unbroken U(1) gauge symmetry, as is verified in the present context in appendix E. On the other hand it has been shown that in the Georgi-Glashow model in 3 dimensions that the so-called "broken" and "unbroken" phases are in fact continuously connected and moreover that no massless photon exists due to condensation of monopoles [9, 10, 27, 28, 29]. It is interesting to note that the last statement agrees with the conclusion of the previous section that excludes a massless gluon pole. This motivates us to examine the consequences of the non-perturbative gauge-fixing of the 5-dimensional formulation when it is adapted to the case of the Georgi-Glashow model. Moreover a consistent definition of a physical particle that is not a singlet under a local gauge group is also difficult in the Higgs phase.

The 5-dimensional action for the Georgi-Glashow model that replaces (3.7) is given by

$$I'_{\rm red} = -\left(\frac{1}{4}\right) \int dt d^4x \left[\left(F_{5\mu} + \frac{\delta S}{\delta A_\mu}\right)^2 + \left(D_5\phi + \frac{\delta S}{\delta \phi}\right)^2 \right], \tag{5.2}$$

where

$$\frac{\delta S}{\delta A_\mu} = -D_\lambda F_{\lambda\mu} + [\phi, D_\mu \phi]\,; \qquad \frac{\delta S}{\delta \phi} = -D_\mu^2 \phi + \lambda(\phi^2 - v^2)\phi\,. \tag{5.3}$$

The second term in this action corresponds to the Langevin equation for ϕ

$$\partial_5\phi = -[A_5, \phi] - \frac{\delta S}{\delta\phi} + \text{noise}\,. \tag{5.4}$$

Here the term $-[A_5, \phi]$ acts as a restoring "force" tangent to the gauge orbit through ϕ.

Their remains to specify A_5 appropriate to this model. If the gauge choice discussed in the preceding section namely, $A_5 = a^{-1}\partial_\mu A_\mu$ in the limit $a \to 0$, is well defined in the present case, then the conclusion of the previous section follows also in the Higgs phase namely that a massless "photon" is excluded. In this context it is helpful to consider a more general class of gauges defined by the more general minimizing functional on the gauge orbit defined by

$$\mathcal{F}_{A,\phi}[g] = \mathcal{F}_{{}^gA,{}^g\phi}[1] = \int d^4x\,\left[(2a)^{-1}|{}^gA|^2 - M\hat{n}\cdot{}^g\phi\right], \tag{5.5}$$

where $t^a({}^g\phi)^a = g^{-1}t^a\phi^a g$. Here the gauge parameters are $a > 0$, $M > 0$, and the direction $\hat{n}$. Because $|{}^g\phi| = |\phi|$, this functional on the gauge orbit is bounded below by $-M\int d^4x\,|\phi|$ which, for given ϕ, is finite for a finite euclidean volume. Clearly, for $M > 0$ this minimizing functional favors configurations ϕ that are aligned along $\hat{n}$, whereas $M = 0$ is a less complete gauge fixing. This minimizing functional offers new possibilities that could be of interest in the context of numerical gauge-fixing in simulations of lattice gauge theory.

We now analyse the gauge fixing associated with this minimizing functional. Under the infinitesimal gauge transformation, $\delta A_\mu = D_\mu\omega$ and $\delta\phi = [\phi, \omega]$, its first and second variations are given by

$$\begin{aligned}\delta\mathcal{F}_{A,\phi}[1] &= -\Big(\omega, a^{-1}\partial_\mu A_\mu + M[\hat{n}, \phi]\Big)\,,\\ \delta^2\mathcal{F}_{A,\phi}[1] &= -\Big(\omega, a^{-1}\partial_\mu D_\mu(A)\omega + M[\hat{n}, [\phi, \omega]]\Big)\,.\end{aligned} \tag{5.6}$$

For A_5 we choose the direction of steepest descent of $\mathcal{F}_{A,\phi}[1]$, restricted to directions tangent to the gauge orbit,

$$\vec{A_5} = a^{-1}\partial_\mu\vec{A}_\mu + M\hat{n}\times\vec{\phi}\,, \tag{5.7}$$

where $\vec{a}\times\vec{b} \equiv [a, b]$. A semi-classical analysis of the action (5.2) with A_5 given in (5.7), is presented in appendix E, which gives the standard semi-classical result namely a pair of charged massive gauge particles and a massless photon associated with the unbroken U(1) symmetry.

For the non-pertubative analysis we consider the gauge defined by (5.7) for large (positive) values of the gauge parameters a^{-1} and M. We scale $M = a^{-1}M'$, and take a to be arbitrarily small. The argument of the preceding section may be used, with the conclusion that in the limit $a \to 0$, the probability gets concentrated near the minima of the minimizing functional $\mathcal{F}_{A,\phi}[g]$, eq. (5.5), namely (i) where $\mathcal{F}_{A,\phi}[g]$ is stationary,

$$\partial_\mu \vec{A}_\mu + M' \hat{n} \times \vec{\phi} = 0 \,, \tag{5.8}$$

and *in addition* (ii) where its second variation is positive namely, by (5.6),

$$\Big(\omega, -\partial_\mu D_\mu(A)\omega - M'\hat{n} \times (\vec{\phi} \times \omega)\Big) \geq 0 \,, \qquad \text{for} \quad \text{all}\ \omega \,. \tag{5.9}$$

The second condition, which expresses the positivity of the relevant Faddeev-Popov operator, is a new, non-perturbative condition, not available in the 4-dimensional formulation, that expresses the restriction to the Gribov region appropriate to this gauge fixing. We note that both conditions are linear in the fields A_μ and ϕ. As a result the Gribov region is convex in A-ϕ space: if $(A^{(i)}, \phi^{(i)})$ lie in the Gribov region for $i = 1, 2$, then (A, ϕ) also lies in the Gribov region for $A = \alpha A^{(1)} + \beta A^{(2)}$ and $\phi = \alpha\phi^{(1)} + \beta\phi^{(2)}$, where $\alpha > 0$ and $\beta = 1 - \alpha > 0$.

Upon taking the vacuum expectation value of (5.8), one obtains

$$M' \ \hat{n} \times \langle \vec{\phi} \rangle = 0 \,, \tag{5.10}$$

so for M' finite, the Higgs field $\vec{\phi}$ cannot acquire a vacuum expectation-value in the direction perpendicular to the gauge parameter $\hat{n}$, whereas no such restriction holds at $M' = 0$. Indeed without an $\hat{n}$ dependence of the gauge-fixing drift force (5.4), there are random walks of the Higgs field in the flat valley. This suggests that $M' = 0$ or $M = 0$ may be a singular point where the gauge is not well defined. However if the gauge $M' = 0$ is well defined, then the Landau-gauge condition $\partial_\mu A_\mu = 0$ holds here as in the previous section which, as we have seen, excludes a massless gauge particle in agreement with Polyakov's conclusion [10]. (We again recall that the position of poles in a propagator is independent of the gauge parameters by virtue of the Nielsen identities.) Note that the Landau gauge is ill-defined in 4-dimensional perturbative calculations because it induces spurious double poles in the propagator of Goldstone bosons.

In terms of the shifted Higgs field, $\vec{\phi} = \vec{v} + \vec{\phi}'$, where $\langle \vec{\phi} \rangle = \vec{v}$, the positivity condition reads

$$\Big(\omega, -\partial_\mu D_\mu(A)\omega - M'\hat{n} \times (\vec{\phi}' \times \omega) + M'v(\omega - \hat{n}\hat{n}\cdot\omega)\Big) \geq 0 \,, \qquad \text{for} \quad \text{all}\ \omega \,. \tag{5.11}$$

The neutral component of $\vec{A}_\mu$ (i. e. along the $\hat{n}$-direction) is restricted only by the components of ω that are perpendicular to $\hat{n}$. So for the neutral component the positivity condition is expressed by

$$\Big(\omega, -\partial_\mu D_\mu(A)\omega - M'\hat{n} \times (\vec{\phi}' \times \omega) + M'v\ \omega\Big) \geq 0 \,, \qquad \omega \perp \hat{n} \,. \tag{5.12}$$

The last term is strictly positive for $M' > 0$, so the restriction on the neutral component is qualitatively weaker than in the $M' = 0$ case and may not be incompatible with a massless gauge particle. The gauge condition now depends on the parameters of the Higgs sector of the model, such as v, as well as on the gauge parameters, such as M, a and $\hat{n}$. Depending on the values of these parameters, the restriction to the Gribov region may give valuable information about the phase such as the position of poles of propagators. This information could be obtained from calculation of propagators by numerical simulation and minimization of a lattice analog of the minimizing functional (5.5) such as

$$\mathcal{F}_{U,\phi}[g] = \mathcal{F}_{{}^gU,{}^g\phi}[1] = \sum_x \; [\, -\, 2a^{-1} \sum_\mu \mathrm{Re}\; \mathrm{tr}\, {}^gU_{x,\mu} - M\hat{n} \cdot {}^g\phi_x \,]\,, \tag{5.13}$$

in the notation of [30].

6. Conclusion

As an alternative to the geometric method presented in [19], in the present article we derived the bulk or stochastic quantization of a gauge field in a series of intuitive steps. The starting point is the bulk quantization of fields of non-gauge type presented in the preceding article [1]. Whereas the standard Faddeev-Popov method relies on *gauge-fixing* that is subject to the problem of Gribov copies, in the step-by-step construction, gauge-fixing is replaced by an A-dependent *gauge transformation* whose jacobian is an infinite constant that cancels the divergent volume of the gauge group. We have shown the perturbative equivalence of the 4- and 5-dimensional formulations of gauge theories by showing that in Landau gauge the Schwinger-Dyson equations of the 4-dimensional theory hold on a time slice of the 5-dimensional theory. We refer to the preceding article [1] for a discussion of the S-matrix that could be formally applied to the case of gauge theories treated perturbatively.

As for physical applications, we have shown that in the limiting case of large gauge parameters, bulk quantization of gauge fields automatically restricts the probability to the interior of the Gribov region in the context of a local, renormalizable theory. For the case of a pure gauge theory, this excludes the existence of physical massless gauge quantum, a first step toward proving confinement. A new result is a minimizing functional (5.13) which is appropriate to global gauge fixing in the presence of coupling to a Higgs field, for which we have found the corresponding Gribov region. The lattice analog of this minimizing functional (5.13) may be used for numerical gauge fixing in simulations of lattice gauge theory.

In this connection we wish to emphasize that lattice discretization of the 5-dimensional theory [30] offers distinct computational possibilities from Monte Carlo simulations of the lattice discretization of the 4-dimensional theory using detailed balance. Discretization of the 5-dimensional theory corresponds to simulation of the

Langevin equation with time-step $\epsilon \sim a^2$, and it is sufficient that they agree in the limit $a \to 0$ [31, 32]. These studies and others [33, 34] have addressed the question of whether the 4- and 5-dimensional discretizations of gauge theories fall into the same universality class and have shown that they do, to first order in ϵ. The present approach, in which the renormalizability of the local 5-dimensional formulation of a gauge theory is assured [19], provides an affirmative answer to this question to all orders.

Acknowledgments

It is a pleasure to thank P. A. Grassi for valuable discussions. The research of Daniel Zwanziger was partially supported by the National Science Foundation under grant PHY-9900769.

A. Continuation of section 2: steps 4 and 5

A.1 Step 4: BRST implementation of the 5-dimensional gauge invariance

The most expedient way to preserve the 5-dimensional gauge symmetry (2.4) and (2.13) is to encode it in a second operator BRST operator w that generates an infinitesimal gauge transformation in the usual way,

$$\begin{aligned} wA_\mu &= D_\mu \lambda \,, \qquad wA_5 = D_5 \lambda \,, \\ w\lambda &= -\frac{1}{2}[\lambda, \lambda] \,, \end{aligned} \tag{A.1}$$

and satisfies $w^2 = 0$. The new Fermi ghost field λ reminds us of the familiar Faddeev-Popov ghost.

We require that the two BRST operators s and w be algebraically consistent in the sense that

$$s^2 = w^2 = sw + ws = 0 \tag{A.2}$$

holds. We also want to construct an action that is both s- and w-invariant and that is physically equivalent to (2.19). We shall modify the action $\hat{I}_{\rm YM}$ by additional ghost terms that involve additional ghost fields, and we shall show in the following section that the action we obtain is physically equivalent to $\hat{I}_{\rm YM}$.

The principle that we use to construct a consistent algebra for s and w is that s should act trivially in the sense that it acts on an elementary field to produce an elementary field rather than a composite. Accordingly we put λ into a new quartet $(\lambda, \mu, \bar{\mu}, \bar{\lambda})$ within which s acts trivially as before, and the action of s on all fields is given in eq. (2.20).

Here μ is a new scalar Bose ghost field that is the topological ghost of the ghost λ, and $\bar{\lambda}$ and $\bar{\mu}$ are the corresponding anti-ghosts. In general we use the "bar" to indicate the anti-ghost of the corresponding ghost, which is also its canonical momentum density, except for $\bar{m}$, which is the anti-ghost of ψ_5, but is not its canonical momentum density in the sense that it enforces a constraint. The last line is a new quartet $(\omega, \phi, \bar{\phi}, \bar{\omega})$ that will be introduced below.

As regards algebraic consistency, we may assign as convenient the w-transform of any of the above elementary fields that is not an s-transform, provided only that it is consistent with $w^2 = 0$. The action of w on any of the above elementary fields that is an s-transform is then determined by the consistency condition $sw + ws = 0$.

We have already stated the w-transforms of the fields A_ν, A_5 and λ. Accordingly the w-transforms of their s-transforms $\psi_\nu = sA_\nu$, $\psi_5 = sA_5$, and $\mu = s\lambda$ are determined by algebraic consistency, namely,

$$\begin{aligned} w\psi_\nu &= wsA_\nu = -swA_\nu = -sD_\nu\lambda = -[\lambda, \psi_\nu] - D_\nu\mu\,, \\ w\psi_5 &= wsA_5 = -swA_5 = -sD_5\lambda = -[\lambda, \psi_5] - D_5\mu\,, \\ w\mu &= ws\lambda = -sw\lambda = \frac{1}{2}s[\lambda, \lambda] = \frac{1}{2}([\mu, \lambda] - [\lambda, \mu]) = -[\lambda, \mu]\,. \end{aligned} \tag{A.3}$$

We now turn to the anti-ghosts $\bar{\psi}_\nu$, $\bar{m}$ and $\bar{\mu}$ that are not the s-transforms of anything. It will be useful for the construction of an s and w-invariant action to assign them the transformation law

$$w\bar{\psi}_\nu = -[\lambda, \bar{\psi}_\nu]\,; \qquad w\bar{\mu} = \bar{m}\,; \qquad w\bar{m} = 0 \tag{A.4}$$

which is consistent with $w^2 = 0$. The w-transforms of their s-transforms $\pi_\nu = s\bar{\psi}_\nu$, $\bar{\lambda} = s\bar{\mu}$, and $l = s\bar{m}$ are determined by algebraic consistency,

$$\begin{aligned} w\pi_\nu &= ws\bar{\psi}_\nu = -sw\bar{\psi}_\nu = s[\lambda, \bar{\psi}_\nu] = -[\lambda, \pi_\nu] + [\mu, \bar{\psi}_\nu]\,; \\ w\bar{\lambda} &= ws\bar{\mu} = -sw\bar{\mu} = -s\bar{m} = -l\,; \\ wl &= ws\bar{m} = -sw\bar{m} = 0\,. \end{aligned} \tag{A.5}$$

One may verify that $w^2 = 0$ is maintained. We have now determined the action of w on all quartets appearing in (2.20) except the last one, which will be determined below, with the result given in (2.21).

Because w generates an infinitesimal gauge transformation on A_μ and A_5, the fields $F_{5\mu}$ and $D_\lambda F_{\lambda\mu}$ transform gauge covariantly, $wF_{5\mu} = -[\lambda, F_{5\mu}]$ and $wD_\mu F_{\mu\nu} = -[\lambda, D_\mu F_{\mu\nu}]$. The anti-ghost field $\bar{\psi}_\nu$ was chosen to also transform gauge-covariantly $w\bar{\psi}_\nu = -[\lambda, \bar{\psi}_\nu]$, so the first term of the s-exact action (2.19),

$$\begin{aligned} I_F &\equiv \int d^5x\; s[\; \bar{\psi}_\mu(F_{5\mu} - D_\lambda F_{\lambda\mu})\;]\,, \\ I_F &= \int dt\; d^4x\; \Big[\pi_\mu\Big(F_{5\mu} - D_\lambda F_{\lambda\mu}\Big) - \bar{\psi}_\mu\Big(D_5\psi_\mu - D_\mu\psi_5 - D_\lambda(D_\lambda\psi_\mu - D_\mu\psi_\lambda) - \\ &\qquad\qquad - [\psi_\lambda, F_{\lambda\mu}]\Big)\Big]\,, \end{aligned} \tag{A.6}$$

is w-invariant, $wI_F = 0$, where I_F is written explicitly below. In fact it is in the cohomology of w, because it is not w-exact, $I_F \neq wX$.

JHEP08(2001)015

To impose the gauge conditions $aA_5 = \partial_\mu A_\mu$ and $a\psi_5 = \partial_\mu \psi_\mu$ in a way which is consistent with both s and w invariance, we take instead of (2.16) the gauge-fixing action,

$$
\begin{aligned}
I_{\rm gf} &\equiv \int d^5x \ sw[\ \bar{\mu}(aA_5 - \partial_\nu A_\nu)]\,, \\
I_{\rm gf} &= \int d^5x \ \Big[l(aA_5 - \partial_\nu A_\nu) - \bar{m}(a\psi_5 - \partial_\nu \psi_\nu) + \bar{\lambda}(aD_5\lambda - \partial_\nu D_\nu \lambda) + \\
&\qquad + \bar{\mu}\Big(\ aD_5\mu - \partial_\nu D_\nu \mu + a[\psi_5, \lambda] - \partial_\nu[\psi_\nu, \lambda]\ \Big)\Big]\,; \qquad \text{(A.7)}
\end{aligned}
$$

that is both s- and w-exact. The first two terms agree with the action $\hat{I}_{\rm gf}$, eq. (2.16), which imposes the desired constraints. With $a > 0$, the remaining terms in the action provide parabolic field equations for the new ghosts λ and μ.

A.2 Step 5: construction of w-covariant fields

Having chosen the transformation law of $\bar{\psi}_\nu$ to be covariant under w, it is inevitable that $\pi_\nu = s\bar{\psi}_\nu$ does not transform covariantly under w, as one sees from (A.5). As a result, the last term of (2.12), $\int d^5x\ s(\bar{\psi}_\nu \pi_\nu)$ is *not* w-invariant. One way to overcome this difficulty is to replace π_ν by $\pi^\star$ defined in (2.23). Here ω is some Fermi-ghost field whose transformation law under w must be such that $\pi^\star_\nu$ is gauge covariant,

$$
w\pi^\star_\nu = -[\lambda, \pi^\star_\nu]\,, \qquad \text{(A.8)}
$$

so that the action,

$$
\begin{aligned}
I_\pi &\equiv \int d^5x \ s(\bar{\psi}_\mu \pi^\star_\mu)\,, \\
I_\pi &= \int d^5x \ s\Big(\bar{\psi}_\mu(\pi_\mu + [\omega, \bar{\psi}_\mu])\Big) = \int d^5x \ \Big(\pi_\mu\pi_\mu + 2\pi_\mu[\bar{\psi}_\mu, \omega] + [\bar{\psi}_\mu, \bar{\psi}_\mu]\phi\Big)\,, \qquad \text{(A.9)}
\end{aligned}
$$

is both s-exact and w-invariant, $wI_\pi = 0$. One easily verifies that $\pi^\star_\nu$ does transform gauge-covariantly, provided that ω satisfies the transformation law $w\omega = -[\lambda, \omega] - \mu$ that appears in the last line of (2.21). It is consistent with $w^2\omega = 0$.

None of the fields we have introduced so far have this transformation law, so we take ω to be a new elementary field. It might be called an "adjuster" field because it allows us to "adjust" π_ν to make a new field that transforms covariantly. To maintain the trivial action of the s-operator, we take the new field ω to be part of a new quartet $(\omega, \phi, \bar{\phi}, \bar{\omega})$ within which s acts as shown in the last line of (2.20). The action of w on ϕ is determined by

$$
w\phi = ws\omega = -sw\omega = s([\lambda, \omega] + \mu) = [s\lambda, \omega] - [\lambda, s\omega] = -[\lambda, \phi] + [\mu, \omega]\,. \qquad \text{(A.10)}
$$

We also assign $\bar{\phi}$ to be w-covariant, which also determines $w\bar{\omega}$. These relations are shown in (2.21). The action (A.9) is s-exact and in the cohomology of w, $wI_\pi = 0$.

We require an action I_ω to provide equations of motion for the new quartet $(\omega, \phi, \bar{\phi}, \bar{\omega})$ that should also be s-exact and w-invariant. To find it, observe that the field ω also allows us to "adjust" the ghost fields ψ_μ and ψ_5, so the adjusted fields (2.24) are w-covariant,

$$w\psi_\mu^\star = -[\lambda, \psi_\mu^\star]\,, \qquad w\psi_5^\star = -[\lambda, \psi_5^\star]\,, \tag{A.11}$$

as is easily verified. The combination $D_\nu \psi_\nu^\star$ is also gauge covariant. Because $\bar{\phi}$, transforms w-covariantly, $w\bar{\phi} = -[\lambda, \bar{\phi}]$, the s-exact action,

$$\begin{aligned} I_\omega &\equiv \int d^5x\ s[\bar{\phi}(a'\psi_5^\star - D_\mu \psi_\mu^\star)] \\ &= \int d^5x\ s\{\bar{\phi}[a'(\psi_5 - D_5\omega) - D_\mu(\psi_\mu - D_\mu\omega)]\} \\ &= \int d^5x\ \Big[\bar{\omega}\Big(- (a'D_5 - D_\nu D_\nu)\omega + a'\psi_5 - D_\nu\psi_\nu\Big) + \\ &\qquad\qquad + \bar{\phi}\Big(- (a'D_5 - D_\nu D_\nu)\phi - a'[\psi_5, \omega] + [\psi_\nu, D_\nu\omega] + D_\nu[\psi_\nu, \omega] - \\ &\qquad\qquad\qquad - [\psi_\nu, \psi_\nu]\Big)\Big]\,, \end{aligned} \tag{A.12}$$

is w-invariant, $wI_\omega = 0$. It provides parabolic equations of motion for ω and ϕ, as long as the otherwise arbitrary parameter a' is positive, $a' > 0$. The total action

$$I \equiv I_F + I_\pi + I_\omega + I_{\rm gf} \tag{A.13}$$

is given in (2.25). This completes the step-by-step construction of the TQFT for a gauge theory.

B. Jacobian of gauge transformation

As announced in section 2, we must check that the jacobian J of the transformation $A \to {}^gA$ is A-independent, $J = \text{const}$. It is sufficient to do this for the infinitesimal gauge transformation that changes the gauge parameter $\alpha \equiv a^{-1}$ by an infinitesimal amount ϵ. Let us determine the infinitesimal gauge transformation $\delta A_\mu = D_\mu(A)\omega$ that achieves this. Assume that $A_5 = \alpha\partial_\mu A_\mu$, and that $A'_5 = (\alpha + \epsilon)\partial_\mu A'_\mu$, where $A'_5 = A_5 + D_5\omega$ and $A'_\mu = A_\mu + D_\mu\omega$, and $\omega = O(\epsilon)$. To first order in ϵ, the last two equations give the condition on ω,

$$\begin{aligned} D_5\omega - \alpha\partial_\mu D_\mu\omega &= \epsilon\partial_\mu A_\mu\,, \\ \partial_5\omega - \alpha D_\mu\partial_\mu\omega &= \epsilon\partial_\mu A_\mu\,. \end{aligned} \tag{B.1}$$

This is a linear, inhomogeneous, parabolic equation for ω. It has the unique solution

$$\omega^a(x, t, A) = \epsilon \int_{-\infty}^t du\ d^4y\ G^{ab}(x, t; y, u; A)\ \partial_\lambda A_\lambda^b(y, u)\,, \tag{B.2}$$

where G is the Green's function defined by

$$(\partial_5 - \alpha D_\mu \partial_\mu) G(x,t;\ y,u) = \delta(x-y)\delta(t-u)\,. \qquad \text{(B.3)}$$

We now calculate the jacobian of the infinitesimal transformation $A'_\mu = A_\mu + D_\mu(A)\omega$. For an infinitesimal transformation with discrete variables, $x'_i = x_i + \epsilon f_i(x)$, say, the jacobian is given by $J = 1 + \epsilon \partial f_i/\partial x_i$, where the second term is a divergence. Thus the jacobian which we must evaluate is given by $J = 1 + K$, where K is the functional trace,

$$K = \int dt\ d^4x \frac{\delta (D_\mu \omega)^a(x,t)}{\delta A^a_\mu(y,u)}|_{y=x,u=t}\,. \qquad \text{(B.4)}$$

To evaluate the functional derivative, consider the variation induced in $(D_\mu\omega)^a(x,t)$ by an infinitesimal variation $\delta A^a_\mu(y,u)$

$$\delta(D^{ac}_\mu \omega^c) = D^{ac}_\mu \delta\omega^c + f^{abc}\delta A^b_\mu \omega^c\,. \qquad \text{(B.5)}$$

Because of the anti-symmetry of the structure constants, the second term does not contribute to the trace, and it is sufficient to consider the variation $D^{ac}_\mu \delta\omega^c$. With ω given in (B.2), we have

$$\begin{aligned}\delta\omega^a(x,t) = \epsilon \int_{-\infty}^t du\ d^4y [G^{ab}(x,t;y,u;A)\ \partial_\lambda \delta A^b_\lambda(y,u) + \\ + \ \delta G^{ab}(x,t;y,u;A)\ \partial_\lambda A^b_\lambda(y,u)]\,. \end{aligned}\qquad \text{(B.6)}$$

We will use the following properties of G:

$$\begin{aligned}G^{ab}(x,t;z,v;A) &= \delta^{ab} G_0(x-z,t-v) + \int_v^t dud^4y G_0(x-y,t-u)\alpha f^{acd} \times \\ &\quad \times\ A^c_\lambda(y,u)\partial_\lambda G^{db}(y,u;z,v;A)\,, \qquad \text{(B.7)}\\ \delta G^{ab}(x,t;z,v;A) &= \int_v^t du\ d^4y\ G^{ac}(x,t;y,u;A)\alpha f^{cde}\delta A^d_\lambda(y,u)\partial_\lambda \times \\ &\quad \times\ G^{eb}(y,u;z,v;A)\,, \qquad \text{(B.8)}\end{aligned}$$

where $G_0(x,t)$ is the free Green function,

$$\begin{aligned}(\partial_t - \alpha\partial_\lambda\partial_\lambda)G_0(x,t) &= \delta(t)\ \delta^4(x)\,,\\ G_0(x,t) &= \theta(t)\left(\frac{a}{4\pi t}\right)^2 \exp\left(-\frac{ax^2}{4t}\right)\,. \end{aligned}\qquad \text{(B.9)}$$

Since we will take the trace, it is sufficient to evaluate $\delta\omega(x,t)$ for variations $\delta A^a_\mu(y,u)$ for u close to t, which greatly simplifies the calculation. Indeed, for u close to t we have

$$G^{ab}(x,t;y,u;A) \approx \delta^{ab} G_0(x-y,t-u)\,, \qquad \text{(B.10)}$$

because the range of the time integration in the second term of (B.7) is negligible. For the same reason, for variations $\delta A^d_\mu(y,u)$ which are non zero only for u close to t, we have $\delta G^{ab}(x,t;z,v;A) \approx 0$, by eq. (B.8). For these variations we may replace (B.6) by its approximate expression

$$\delta\omega^a(x,t) \approx \epsilon \int_{-\infty}^{t} du\ d^4y\ G_0(x-y,t-u)\ \partial_\lambda \delta A^a_\lambda(y,u)\,. \tag{B.11}$$

With this result, we obtain for the required variation $\delta(D^{ac}_\mu \omega^c) = D^{ac}_\mu \delta\omega^c$

$$\delta(D^{ac}_\mu \omega^c) = \epsilon\ D^{ac}_\mu \int_{-\infty}^{t} du\ d^4y\ G_0(x-y,t-u)\ \partial_\lambda \delta A^c_\lambda(y,u)\,. \tag{B.12}$$

To evaluate K which is the trace, eq. (B.4), we need only the diagonal part of the variation, so by the anti-symmetry of f^{abc} we may replace this by

$$\delta(D^{ac}_\mu \omega^c) = -\epsilon\ \partial_\mu \int_{-\infty}^{t} du\ d^4y\ G_0(x-y,t-u)\ \partial_\lambda \delta A^a_\lambda(y,u)\,. \tag{B.13}$$

The coefficient of $\delta A^a_\lambda(y,u)$ is independent of A. Consequently K is independent of A, and thus so is the jacobian $J = 1 + K$. Thus J is a (divergent) constant as asserted. The demonstration relied heavily on the retarded properties of the Green function of parabolic operators.

C. Equivalence of standard and bulk quantization for gauge theories

In this appendix we shall revisit the proof that, perturbatively, the Faddeev-Popov formulation in 4 dimensions gives the same result as the present 5-dimensional formulation for gauge-invariant observables. To do this we shall show that the two formulations give the same correlation functions — including gauge non-invariant ones — in the Landau-gauge limit, $a \to 0$. Because both formulations are gauge-parameter independent for gauge-invariant quantities, this establishes the perturbative equivalence of the two theories. The limit $a \to 0$ is delicate in the 5-dimensional formulation because some propagators become elliptic instead of parabolic, and individual Feynman diagrams may be singular at $a = 0$. However we expect that the correlation functions remain finite in this limit. Technically one can compare this limit to the one encountered when one regularizes the singular behavior of the Coulomb gauge in 4 dimensions by a renormalizable gauge $\xi\partial_0 A_0 + \partial_i A_i = 0$, with $\xi \to 0$ [35].

Consider a generic functional $\mathcal{O} = \mathcal{O}[A]$ of the 4-dimensional gauge theory, not necessarily gauge invariant, whose expectation-value we wish to compute. It is sufficient to take $\mathcal{O} = \exp(J,A)$, in which case the expectation-value $\langle\mathcal{O}\rangle = Z(J)$ is the generating functional of all correlation functions. In the perturbative 4-dimensional formulation, $\langle\mathcal{O}\rangle$ is computed using the path integral weighted by the exponential of the Faddeev-Popov action

$$S_{\rm FP} = \int d^4x \ \left[\left(\frac{1}{4}\right) F_{\mu\nu}^2 - \partial_\mu\bar\eta D_\mu\eta + \partial_\mu h A_\mu + M h^2\right]. \tag{C.1}$$

This action is invariant under the ordinary BRST-invariance of the Faddeev-Popov theory $\underline{s}S_{\rm FP} = 0$, where $\underline{s}A_\mu = D_\mu\eta,\ \underline{s}\eta = -\eta^2,\ \underline{s}\bar\eta = h,\ \underline{s}h = 0$. From the identity, $0 = \int d\Phi \ \frac{\delta}{\delta A_\mu}[\mathcal{O}\exp(-S_{\rm FP})]$, we obtain order by order in perturbation theory, the following Schwinger-Dyson (SD) equation:

$$\left\langle \frac{\delta\mathcal{O}}{\delta A_\mu} - \mathcal{O}(-D_\lambda F_{\lambda\mu} + [\partial_\mu\bar\eta,\eta] + \partial_\mu h)\right\rangle_{\rm FP} = 0\,, \tag{C.2}$$

where the mean value is computed perturbatively from the Faddeev-Popov action (C.1). We next write $h = \underline{s}\bar\eta$, and use BRST-invariance to reexpress the the last term,

$$\langle\mathcal{O}\ \partial_\mu h(x)\rangle_{\rm FP} = \langle\mathcal{O}\ \underline{s}\partial_\mu\bar\eta(x)\rangle_{\rm FP} = -\int d^4y \left\langle \frac{\delta\mathcal{O}}{\delta A_\lambda(y)}(D_\lambda\eta)(y)\ \partial_\mu\bar\eta(x)\right\rangle_{\rm FP}\,, \tag{C.3}$$

so the SD equation reads

$$\left\langle \frac{\delta\mathcal{O}}{\delta A_\lambda}(x) + \int d^4y\partial_\mu\bar\eta(x)\eta(y)D_\lambda\frac{\delta\mathcal{O}}{\delta A_\lambda}(y) - \mathcal{O}(-D_\lambda F_{\lambda\mu} + [\partial_\mu\bar\eta,\eta])(x)\right\rangle_{\rm FP} = 0\,. \tag{C.4}$$

Finally, we integrate out the Faddeev-Popov ghosts, and use $\langle\bar\eta(x)\eta(y)\rangle = M^{-1}(x,y; A)$, where $M(A) = -D_\mu(A)\partial_\mu$ is the (hermitean conjugate of the) Faddeev-Popov operator, which gives

$$\left\langle (I + \partial M^{-1}D)_{\mu\lambda}\frac{\delta\mathcal{O}}{\delta A_\lambda} - \mathcal{O}(-D_\lambda F_{\lambda\mu} + \partial_\mu M^{-1}\right\rangle_{\rm FP} = 0\,, \tag{C.5}$$

where is last term is given explicitly by $(\partial_\mu M^{-1})^a(x) \equiv f^{abc}\partial_\mu(M^{-1})^{bc}(x,y)|_{y=x}$.

We now show that this equation holds in the 5-dimensional theory by a generalization of the theory of non-gauge type discussed in [1]. We start with the identity

$$\int d\Phi\frac{\delta}{\delta\pi(x)}\Big(\mathcal{O}[A]\exp I\Big) = 0\,. \tag{C.6}$$

Here the integral is over all fields of the 5-dimensional theory, with action (2.25), but the observable $\mathcal{O}[A]$ depends only on $A_\mu = A_\mu(x_\lambda,0)$ at $t = x_5 = 0$. This is a generic physical observable, and coincides wtih the observable in eq. (C.2). From the action (2.25) one obtains

$$\big\langle\mathcal{O}\ (F_{5\mu} - D_\lambda F_{\lambda\mu} + 2[\omega,\bar\psi_\mu] + 2\pi_\mu)(x)\big\rangle_{\rm TQFT_5} = 0\,, \tag{C.7}$$

where the argument $x = (x_\lambda, 0)$ is also at $t = x_5 = 0$.

We shall show that in the Landau gauge, $a = 0$, this equation reduces to the form (C.5). In appendix C, it is proven that in this gauge, $F_{5\mu}$ is odd under time-reversal, $F_{5\mu}(x_\lambda, 0) \to -F_{5\mu}(x_\lambda, 0)$, whereas $\mathcal{O}[A]$ is even, $\mathcal{O}[A] \to \mathcal{O}[A]$, for quantities $\mathcal{O}[A]$ that depend only on $A_\mu = A_\mu(x_\lambda, 0)$ at $t = 0$. As a result the first correlator in (C.7) vanishes, $\langle \mathcal{O}[A]\ F_{5\mu} \rangle_{TQFT_5} = 0$.

We next write $\pi_\mu = s\bar{\psi}_\mu$, and use s-invariance to rewrite the the last term,

$$\langle \mathcal{O}\ \pi_\mu(x) \rangle_{\mathrm{FP}} = \langle \mathcal{O}\ s\bar{\psi}_\mu(x) \rangle_{\mathrm{TQFT}_5} = -\int d^4y \left\langle \frac{\delta \mathcal{O}}{\delta A_\lambda(y)} \psi_\lambda(y)\ \bar{\psi}_\mu(x) \right\rangle_{\mathrm{TQFT}_5} . \tag{C.8}$$

Here the $t = x_5$ component of $y = (y_\lambda, 0)$ vanishes because $\mathcal{O}[A]$ only depends upon A at $t = 0$. This gives

$$\left\langle 2 \int d^4y\ \bar{\psi}_\mu(x)\ \psi_\lambda(y) \frac{\delta \mathcal{O}}{\delta A_\lambda}(y) + \mathcal{O}(-D_\lambda F_{\lambda\mu} + 2[\omega, \bar{\psi}_\mu])(x) \right\rangle_{\mathrm{TQFT}_5} = 0\,. \tag{C.9}$$

We now evaluate the equal-time ghost propagators that appear here in terms of the A-field. Although the action contains cubic ghost terms nevertheless, because the ghost action is parabolic, the ghost-field propagators at equal time do not depend on the interaction and may be evaluated exactly. To evaluate them we expand the action (2.25) and obtain

$$\begin{aligned} I = \int d^5x \Big[\cdots - \bar{\psi}_\mu(D_5\psi_\mu - D_\mu\psi_5 + \cdots) + \bar{\omega}[a'(\psi_5 - D_5\omega) + \cdots] \\ + l(aA_5 - \partial_\mu A_\mu) - \bar{m}(a\psi_5 - \partial_\mu\psi_\mu) + \cdots \Big] , \end{aligned} \tag{C.10}$$

where we have used (A.6), (A.7) and (A.12).

We now set $a = 0$ in this expression. The variables A_5 and ψ_5 are no longer constrained by the gauge condition. Integration on $\bar{m}$ and ψ_5 respectively imposes the constraints $\partial_\mu\psi_\mu = 0$, and $a'\bar{\omega} = D_\mu\bar{\psi}_\mu$. In terms of the remaining variables ψ_μ^{tr}, ω and $\bar{\psi}_\mu = \bar{\psi}_\mu^{\mathrm{tr}} + \partial_\mu\bar{\rho}$, the action (C.10) becomes

$$\begin{aligned} I &= \int d^5x \Big[\cdots - \bar{\psi}_\mu(\partial_5\psi_\mu^{\mathrm{tr}} + \cdots) - D_\mu\bar{\psi}_\mu(\partial_5\omega + \cdots) - l\partial_\mu A_\mu + \cdots \Big] \qquad \text{(C.11)} \\ &= \int d^5x \Big[\cdots - \bar{\psi}_\mu^{\mathrm{tr}}(\partial_5\psi_\mu^{\mathrm{tr}} + \cdots) - (D_\mu\bar{\psi}_\mu^{\mathrm{tr}} + D_\mu\partial_\mu\bar{\rho})(\partial_5\omega + \cdots) - l\partial_\mu A_\mu + \cdots \Big] . \end{aligned}$$

The ghost propagators at equal time are determined by the terms in ∂_5 only. In fact, for a generic parabolic action of the form $\int dtd^4x(L_{ij}\bar{\sigma}_j\partial_5\sigma_i + \cdots)$, where L is a time-independent linear operator, the equal-time propagator is given by $\sigma(x)\bar{\sigma}(y) = (1/2)L^{-1}(x, y)$. As a result we have $\langle \psi_\lambda(x)\bar{\psi}_\mu(y) \rangle_{\mathrm{TQFT}_5} = (1/2)(I + DM^{-1}\partial)$, and $\langle [\omega(x), \bar{\psi}_\mu(y)] \rangle_{\mathrm{TQFT}_5} = (1/2)\partial_\mu M^{-1}(x, y)$, where quantities are defined as in eq. (C.5). With the substitution of these values, eq. (C.9) of the 5-dimensional theory agrees with the 4-dimensional SD equations (C.5).

We have established that in the Landau gauge limit of the 5-dimensional theory the SD equations of the 4-dimensional theory are satisfied. This shows that all correlation functions of the A_μ-field agree in this gauge. Note that if we compare (C.1) with (C.11), we find the interesting correspondences in Landau gauge of the 4-dimensional Faddeev-Popov ghost with the bulk quantities: $\eta \leftrightarrow \omega$ and $\bar\eta \leftrightarrow \bar\rho$.

The proof given here is an alternative to the one displayed in [20], still in the context of perturbation theory, which to our knowledge was the only existing one for comparing the predictions of both formulations. That proof relied on a definition of correlation functions as the solution of a Fokker-Planck process that involved a relaxation to equilibrium and some non-local interactions, while the proof we have just given in this paper relies on a local quantum field theory in 5 dimensions that moreover is time-translation invariant. We do not expect that the Faddeev-Popov measure allows one to compute beyond perturbation theory, while, on the other hand, the 5-dimensional formulation is expected to also hold non-perturbatively.

D. Proof of time-reversal invariance

Here we extend the argument of to gauge theories. Consider the w-invariant action

$$I_{\rm tot,w}(a) \equiv I_{\rm w}(a) + \frac{1}{2}\int d^5x\, \frac{\delta S}{\delta A_\mu} F_{5\mu}\,. \tag{D.1}$$

which differs from (3.5) by the second term which is an exact derivative. Indeed we have $F_{5\mu} = \dot A_\mu - D_\mu A_5$ and $\dot S = \int d^4x \frac{\delta S}{\delta A_\mu}\dot A_\mu$, and moreover $D_\mu \frac{\delta S}{\delta A_\mu} = 0$ by gauge invariance of S. All terms in the actions (3.5) and (D.1) are separately w-invariant because both $\pi_\mu^\star$ and $F_{5\mu}$ are w-covariant, S is gauge covariant, and $w^2 = 0$. Upon expansion, the action (3.5) at $a = 0$ reads

$$I_{\rm w}(0) = \int d^5x \left[\pi_\mu^\star\left(F_{5\mu} + \frac{\delta S}{\delta A_\mu} + \pi_\mu^\star\right) - \bar\lambda\partial_\mu D_\mu\lambda + l\partial_\mu A_\mu\right]. \tag{D.2}$$

We shall show that $I_{\rm tot,w}(0)$ is invariant under the time-reversal transformation

$$\begin{aligned}
A_\mu(x,t) &\longrightarrow A_\mu^{\rm T}(x,t) = A_\mu(x,-t)\,,\\
A_5(x,t) &\longrightarrow A_5^{\rm T}(x,t) = -A_5(x,-t)\,,\\
\pi_\mu^\star(x,t) &\longrightarrow \pi_\mu^{\star\rm T}(x,t) = -\pi_\mu^\star(x,-t) - \frac{\delta S}{\delta A_\mu}(x,-t)\,,\\
\lambda(x,t) &\longrightarrow \lambda^{\rm T}(x,t) = \lambda(x,-t)\,,\\
\bar\lambda(x,t) &\longrightarrow \bar\lambda^{\rm T}(x,t) = \bar\lambda(x,-t)\,,\\
l(x,t) &\longrightarrow l^{\rm T}(x,t) = l(x,-t)\,.
\end{aligned} \tag{D.3}$$

In terms of the variables $F_{5\mu}$ and $\pi'_\mu \equiv \pi_\mu^\star + \frac{1}{2}\frac{\delta S}{\delta A_\mu}$ these transformations imply

$$\begin{aligned}
\pi'_\mu(x,t) &\longrightarrow \pi'_\mu T(x,t) = -\pi'_\mu(x,-t)\,,\\
F_{5\mu}(x,t) &\longrightarrow F_{5\mu}^{\rm T}(x,t) = -F_{5\mu}(x,-t)\,,
\end{aligned} \tag{D.4}$$

the action $I_{\rm tot,w}(0)$ reads

$$I_{\rm tot,w}(0) = \int d^5x \Big[\pi'_\mu F_{5\mu} + \pi'^2_\mu - \frac{1}{4}\Big(\frac{\delta S}{\delta A_\mu}\Big)^2 - \bar\lambda\partial_\mu D_\mu\lambda + l\partial_\mu A_\mu\Big] . \tag{D.5}$$

This action is manifestly invariant under the above transformation. Note that the symmetry $t \to -t$ is violated by the w-exact term in the action (3.5) for $a \neq 0$. This is a symmetry of the observables since they are defined as the cohomology of w at $t = 0$.

We have proven that in the Landau gauge, $a = 0$, the action $I_{\rm tot,w}(0)$ is invariant under the time reversal transformation. This is a singular gauge in the 5-dimensional formulation. However we expect that the correlation functions calculated at finite a have a finite limit $a \to 0$ which enjoys this symmetry.

E. Semi-classical analysis of the Higgs phase

We now make a semi-classical analysis of the action (5.2), with A_5 given in (5.7). We shift $\vec\phi$ by $\vec\phi = \vec v + \vec\phi'$, where $\vec v \equiv v\hat n$ has the magnitude v that appears in the classical action (5.1), and $\hat n$ is the direction that appears in the gauge choice (5.7). This gives

$$\begin{aligned}
\vec F_{5\mu} + \frac{\delta S}{\delta \vec A_\mu} &= \partial_5 \vec A_\mu - \partial_\lambda(\partial_\lambda \vec A_\mu - \partial_\mu \vec A_\lambda) - a^{-1}\partial_\mu\partial_\lambda \vec A_\lambda + \vec v \times (\vec A_\mu \times \vec v) + \\
&\quad + (v - M)\hat n \times \partial_\mu \vec\phi' + \text{nonlinear} , \\
D_5\vec\phi + \frac{\delta S}{\delta\vec\phi} &= \partial_5\vec\phi' - \partial^2_\mu\vec\phi' + 2\lambda\vec v\; \vec v\cdot\vec\phi' + M(\hat n \times \vec\phi') \times \vec v + \\
&\quad + (a^{-1} - 1)\partial_\lambda \vec A_\lambda \times \vec v + \text{nonlinear} ,
\end{aligned} \tag{E.1}$$

where have written explicitly only terms that are linear in A_μ and ϕ'. These expressions are substituted into the action $I'_{\rm red}$, eq. (5.2). Note first that $I'_{\rm red}$ is quadratic in $\vec A_\mu$ and $\vec\phi'$ (plus higher order terms), so the classical vacuum is indeed given by $\vec A_\mu = 0$ and $\vec\phi' = 0$. This corresponds to the classical vacuum of $\vec\phi$ being given by $\langle\vec\phi\rangle = \vec v = v\hat n$, where the direction $\hat n$ is the gauge parameter introduced in (5.7). The direction of the vacuum in the Higgs phase is determined by the gauge-fixing.

The last term in (E.1) causes mixing of the would-be Goldstone boson with the longitudinal part of the A_μ field. However for the special gauge defined by $M = v$ and $a = 1$ both mixing terms vanish. In this case the free propagators are given by

$$\begin{aligned}
D^{AA}_{\mu\nu} &= \left(\frac{1}{2}\right) \delta_{\mu\nu}\{P_\pm[\omega^2 + (k^2 + v^2)^2]^{-1} + P_0[\omega^2 + (k^2)^2]^{-1}\} , \\
D^{\phi\phi} &= \left(\frac{1}{2}\right) \{P_\pm[\omega^2 + (k^2 + v^2)^2]^{-1} + P_0[\omega^2 + (k^2 + 2\lambda v^2)^2]^{-1}\} , \\
D^{A\phi} &= 0 ,
\end{aligned} \tag{E.2}$$

where the charged and neutral projectors are $P_{\pm}^{bc} = \delta^{bc} - \hat{n}^b\hat{n}^c$ and $P_0^{bc} = \hat{n}^b\hat{n}^c$. The 4-dimensional propagators are obtained by setting the times equal,

$$D^{(4)}(k) = D(t,k)|_{t=0} = (2\pi)^{-1}\int d\omega D(\omega,k)\,. \tag{E.3}$$

This gives (in the special gauge $M = v$ and $a = 1$),

$$\begin{aligned} D_{\mu\nu}^{(4)AA} &= \delta_{\mu\nu}\{P_{\pm}(k^2+v^2)^{-1} + P_0(k^2)^{-1}\}\,, \\ D^{(4)\phi\phi} &= P_{\pm}(k^2+v^2)^{-1} + P_0(k^2+2\lambda v^2)^{-1}\,, \\ D^{(4)A\phi} &= 0\,. \end{aligned} \tag{E.4}$$

One recognizes the free propagators of the 't Hooft-Feynman gauge in the 4-dimensional formulation. In particular only the neutral gauge particle remains massless, corresponding to the unbroken U(1) symmetry, and the would-be Goldstone boson, namely the charged components of ϕ', acquires a mass v^2. Thus, at the semi-classical level things behave rather like in the 4-dimensional formulation in this gauge, even though, as we have seen, at the non-perturbative level the 5-dimensional formulation of the Higgs phase is quite different from the 4-dimensional Faddeev-Popov formulation because of the additional condition (5.9).

For other values of the gauge parameters there is mixing of the would-be Goldstone bosons with the longitudinal part of A. Indeed the $A\phi$-term in the quadratic part of $I'_{\rm red}$, eq. (5.2), is given, after integration by parts, by

$$\begin{aligned} I_0^{A\phi} = -\left(\frac{1}{2}\right)\int dtd^4x\ (\hat{n}\times\partial_\lambda\vec{A}_\lambda)\,\cdot \\ \cdot\left[(M-a^{-1}v)\partial_5\vec{\phi}' + (v-a^{-1}M)(-\partial_\lambda^2+v^2)\vec{\phi}'\right]. \end{aligned} \tag{E.5}$$

The first term vanishes for $M = a^{-1}v$, and the second for $M = av$, but both vanish only for $M = v$ and $a = 1$. However one does recover the familiar 4-dimensional free propagators for the physical degrees of freedom, namely the transverse A-propagator and the neutral ϕ-propagator.

We also give the form of the free propagators for more general values of the gauge parameters. For $M = v$ and $a \neq 1$, the propagators for the gauge fields and Higgs fields are:

$$\begin{aligned} D_{\mu\nu}^{AA} = \left(\frac{1}{2}\right) P_{\mu\nu}^{\rm tr}\{P_{\pm}[\omega^2+(k^2+v^2)^2]^{-1} + P_0[\omega^2+(k^2)^2]^{-1}\} + \\ + \left(\frac{1}{2}\right) P_{\mu\nu}^{\rm lo}\{P_{\pm}[\omega^2+(k^2+v^2)(a^{-2}k^2+v^2)]^{-1} + P_0[\omega^2+a^{-2}(k^2)^2]^{-1}\}\,, \end{aligned} \tag{E.6}$$

where $P_{\mu\nu}^{\rm tr} \equiv \delta_{\mu\nu} - \hat{k}_\mu\hat{k}_\nu$ and $P_{\mu\nu}^{\rm lo} \equiv \hat{k}_\mu\hat{k}_\nu$, and

$$\begin{aligned} D^{\phi\phi} &= \left(\frac{1}{2}\right)\{P_{\pm}[\omega^2+(k^2+v^2)^2]^{-1} + P_0[\omega^2+(k^2+2\lambda v^2)^2]^{-1}\}\,, \\ D^{\phi^a A_\lambda^b} &= \left(\frac{1}{2}\right)(a^{-1}-1)\ \epsilon^{abc}v^c\ ik_\lambda[-i\omega+k^2+v^2]^{-1}[\omega^2+(k^2+v^2)(a^{-1}k^2+v^2)]^{-1}\,. \end{aligned} \tag{E.7}$$

This gives $D^{(4)}(k) = [(k^2+v^2)(a^{-2}k^2+v^2)]^{-1/2}$ for the charged, longitudinal free A-propagator in 4-dimensions, which does not correspond to any known 4-dimensional gauge.

Another simplifying gauge choice is $M \neq v$, and $a = 1$, for which one obtains

$$\begin{aligned} D^{AA}_{\mu\nu} &= \left(\frac{1}{2}\right) \delta_{\mu\nu} \{ P_{\pm}[\omega^2 + (k^2+v^2)^2]^{-1} + P_0[\omega^2 + (k^2)^2]^{-1} \} \,, \\ D^{\phi\phi} &= \left(\frac{1}{2}\right) \{ P_{\pm}[\omega^2 + (k^2+v^2)(k^2+M^2)]^{-1} + P_0[\omega^2 + (k^2+2\lambda v^2)^2]^{-1} \} \,. \\ D^{A^b_\lambda \phi^c} &= \left(\frac{1}{2}\right) (v - M)\ \epsilon^{bca}\hat{n}^a\ ik_\lambda[-i\omega + k^2 + v^2]^{-1}[\omega^2 + (k^2+v^2)(k^2+M^2)] \end{aligned} \tag{E.8}$$

This gives $D^{(4)}(k) = [(k^2+v^2)(k^2+M^2)]^{-1/2}$ for the 4-dimensional propagator of the would-be Goldstone boson, which again does not correspond to any known 4-dimensional gauge.

References

[1] L. Baulieu and D. Zwanziger, *From stochastic quantization to bulk quantization: Schwinger-Dyson equations and S-matrix*, *J. High Energy Phys.* **08** (2001) 016 [hep-th/0012103].

[2] V.N. Gribov, *Quantization of nonabelian gauge theories*, *Nucl. Phys.* **B 139** (1978) 1.

[3] D. Zwanziger, *Vanishing of zero momentum lattice gluon propagator and color confinement*, *Nucl. Phys.* **B 364** (1991) 127.

[4] A. Cucchieri and D. Zwanziger, *Numerical study of gluon propagator and confinement scenario in minimal coulomb gauge*, hep-lat/0008026.

[5] A. Cucchieri and D. Zwanziger, *Fit to gluon propagator and Gribov formula*, hep-lat/0012024.

[6] N.K. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, *Nucl. Phys.* **B 101** (1975) 173.

[7] D. Zwanziger, *Renormalization in the coulomb gauge and order parameter for confinement in QCD*, *Nucl. Phys.* **B 518** (1998) 237.

[8] T. Kugo and I. Ojima, *Local covariant operator formalism of non-abelian gauge theories and quark confinement problem*, *Suppl. Prog. Theor. Phys.* **66** (1979) 1.

[9] G. 't Hooft, *Magnetic monopoles in unified gauge theories*, *Nucl. Phys.* **B 79** (1974) 276;
Topology of the gauge condition and new confinement phases in nonabelian gauge theories, *Nucl. Phys.* **B 190** (1981) 455.

[10] A.M. Polyakov, *Compact gauge fields and the infrared catastrophe*, *Phys. Lett.* **B 59** (1975) 82; *Quark confinement and topology of gauge groups*, *Nucl. Phys.* **B 120** (1977) 429; *Gauge fields and strings*, ch. 4 Harwood Academic Publishers, 1987.

[11] S. Nadkarni, *The* SU(2) *adjoint Higgs model in three-dimensions*, *Nucl. Phys.* **B 334** (1990) 559.

[12] A. Hart, O. Philipsen, J.D. Stack and M. Teper, *On the phase diagram of the* SU(2) *adjoint Higgs model in* $2+1$ *dimensions*, *Phys. Lett.* **B 396** (1997) 217 [hep-lat/9612021].

[13] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, *3d* SU(N)+ *adjoint Higgs theory and finite-temperature QCD*, *Nucl. Phys.* **B 503** (1997) 357 [hep-ph/9704416].

[14] S. Elitzur, *Impossibility of spontaneously breaking local symmetries*, *Phys. Rev.* **D 12** (1975) 3978.

[15] R.P. Feynman, *The qualitative behavior of Yang-Mills theory in (*$2+1$*)- dimensions*, *Nucl. Phys.* **B 188** (1981) 479.

[16] I.M. Singer, *Some remarks on the gribov ambiguity*, *Comm. Math. Phys.* **60** (1978) 7.

[17] R.E. Cutkosky, *Harmonic functions and matrix elements for hyperspherical quantum field models*, *J. Math. Phys.* **25** (1984) 939;
R.E. Cutkosky and K.C. Wang, *Vacuum and excited states of coulomb gauge* SU(N) *Yang-Mills fields*, *Phys. Rev.* **D 37** (1988) 3024;
R.E. Cutkosky, *Czech. J. Phys.* **40** (1990) 252

[18] J. Koller and P. van Baal, *A nonperturbative analysis in finite volume gauge theory*, *Nucl. Phys.* **B 302** (1988) 1;
P. van Baal, *More (thoughts on) Gribov copies*, *Nucl. Phys.* **B 369** (1992) 259;
P. van Baal and N.D. Hari Dass, *The theta dependence beyond steepest descent*, *Nucl. Phys.* **B 385** (1992) 185.

[19] L. Baulieu, P.A. Grassi and D. Zwanziger, *Gauge and topological symmetries in the bulk quantization of gauge theories*, *Nucl. Phys.* **B 597** (2001) 583 [hep-th/0006036].

[20] L. Baulieu and D. Zwanziger, *Equivalence of stochastic quantization and the Faddeev-Popov ansatz*, *Nucl. Phys.* **B 193** (1981) 163.

[21] G. Parisi and Y.S. Wu, *Sci. Sinica* **24** (1981) 484.

[22] E. Gozzi, *Functional integral approach to parisi-wu stochastic quantization: scalar theory*, *Phys. Rev.* **D 28** (1983) 1922.

[23] J. Zinn-Justin, *Renormalization and stochastic quantization*, *Nucl. Phys.* **B 275** (1986) 135.

[24] D. Zwanziger, *Covariant quantization of gauge fields without gribov ambiguity*, *Nucl. Phys.* **B 192** (1981) 259.

[25] H.S. Chan and M.B. Halpern, *New ghost free infrared-soft gauges*, *Phys. Rev.* **D 33** (1986) 540.

[26] L. Baulieu and B. Grossman, *A topological interpretation of stochastic quantization*, *Phys. Lett.* **B 212** (1988) 351;
L. Baulieu, *Stochastic and topological gauge theories*, *Phys. Lett.* **B 232** (1989) 479; *Topological quantum field theories and gauge invariance in stochastic quantization*, *Int. J. Mod. Phys.* **A 6** (1991) 2793.

[27] K. Osterwalder and E. Seiler, *Gauge field theories on the lattice*, *Ann. Phys. (NY)* **110** (1978) 440.

[28] E.H. Fradkin and S.H. Shenker, *Phase diagrams of lattice gauge theories with Higgs fields*, *Phys. Rev.* **D 19** (1979) 3682.

[29] T. Banks and E. Rabinovici, *Finite temperature behavior of the lattice abelian Higgs model*, *Nucl. Phys.* **B 160** (1979) 349.

[30] L. Baulieu and D. Zwanziger, *QCD_4 from a five-dimensional point of view*, *Nucl. Phys.* **B 581** (2000) 604 [hep-th/9909006].

[31] G.G. Batrouni, G.R. Katz, A.S. Kronfeld, G.P. Lepage, B. Svetitsky and K.G. Wilson, *Langevin simulations of lattice field theories*, *Phys. Rev.* **D 32** (1985) 2736.

[32] C.T.H. Davies, G.G. Batrouni, G.R. Katz, A.S. Kronfeld, G.P. Lepage, K.G. Wilson, P. Rossi and B. Svetitsky, *Fourier acceleration in lattice gauge theories, 3. Updating field configurations*, *Phys. Rev.* **D 41** (1990) 1953.

[33] M. Fukugita, Y. Oyanagi and A. Ukawa, *Hadron spectroscopy in lattice QCD with dynamical quark loops*, *Phys. Rev. Lett.* **57** (1986) 953; *Langevin simulation of the full QCD hadron mass spectrum on a lattice*, *Phys. Rev.* **D 36** (1987) 824.

[34] A.S. Kronfeld, *Dynamics of Langevin simulation Prog. Theor. Phys. Suppl.* **111** (1993) 293.

[35] L. Baulieu and D. Zwanziger, *Renormalizable non-covariant gauges and coulomb gauge limit*, *Nucl. Phys.* **B 548** (1999) 527 [hep-th/9807024].